# A novel approach for the pulsar magnetosphere


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## Abstract

**Context.** Many numerical solutions of the pulsar magnetosphere over the past 25 years show closed-line regions that end a significant distance inside the light cylinder, and manifest thick strongly dissipative separatrix surfaces instead of thin current sheets, with a tip that has a distinct pointed Y shape instead of a T shape. We need to understand the origin of these results which were not predicted by our early theories of the pulsar magnetosphere.

**Aims.** In order to gain new intuition on this problem, we set out to obtain the theoretical steady-state solution of the 2D and 3D ideal force-free magnetosphere with zero dissipation along the separatrix and equatorial current sheets. In order to achieve our goal, we needed to develop a novel numerical method.

**Methods.** We solve two independent magnetospheric problems without current sheet discontinuities in the domains of open and closed field lines, and adjust the shape of their interface (the separatrix) to satisfy pressure balance between the two regions. The solution is obtained with meshless Physics Informed Neural Networks (PINNs).

**Results.** In this poster we present our results for the axisymmetric problem and an inclined(20° deg) dipole rotator using the new methodology. We are able to zoom-in around the Y-point and inside the closed-line region with unprecedented detail, and we observe features that were never been discussed in previous numerical solutions. This is the first time the steady-state 3D problem is addressed directly, and not through a time-dependent simulation that eventually relaxes to a steady-state.


## Mathematical formulation

- $\rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$ *(Force-free equation)*
- $\mathbf{J} = \frac{c}{4\pi} \nabla \cdot \mathbf{E} \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{c}{4\pi} \frac{\mathbf{B} \cdot \nabla \times \mathbf{B} - \mathbf{E} \cdot \nabla \times \mathbf{E}}{B^2} \mathbf{B}$ (Gruzinov(1999) and Blandford (2002))

**All fields are calculated in the non-rotating inertial lab frame**
(We used the approach of Muslimov & Harding (2009))

Fields must satisfy Maxwell's equations. By rewriting these equations in our chosen reference frame, we arrive at the following equation (Endean (1974) and Mestel (1975):

$$\nabla \times \{\mathbf{B}_\rho (1 - (\frac{r \sin\theta}{R_{LC}})^2) + \mathbf{B}_\phi\} = a\mathbf{B}$$

Here, α is a force-free parameter that obeys the extra constraint:

$$\mathbf{B} \cdot \nabla a = 0$$


## REFERENCES

- Gruzinov A., 1999, astro-ph/9902288
- Blandford R., 2002, in Gilfanov M., Sunyaev R., Churazov E., eds, ESO Astroph. Symp. Springer-Verlag, Berlin, p. 381
- Muslimov A., Harding A. K., 2009, ApJ, 692, 140
- Endean V. G., 1974, ApJ, 187, 359
- Mestel L., 1975, Soc. Roy. des Sciences de Leège, 8, 79
- Timokhin A., 2006, MNRAS, 368, 1055
- Spitkovsky A., 2006, ApJ, 648, L51
- Lyubarskii Yu. E., 1990, Soviet Astron. Lett., 16, 16

### Related Works by the Author

- Dimitropoulos, I., Contopoulos, I., Mpisketzis, V., & Chaniadakis, E. 2024, MNRAS, 528, 3141 (Paper I)
- Contopoulos, I., Dimitropoulos, I., Ntotsikas, D., & Gourgouliatos, K. N. 2024, Universe, 10, 178 (Paper II)
- Dimitropoulos, I. Contopoulos, I., Chanadiakis, E. 2025, arXiv:2410.10716



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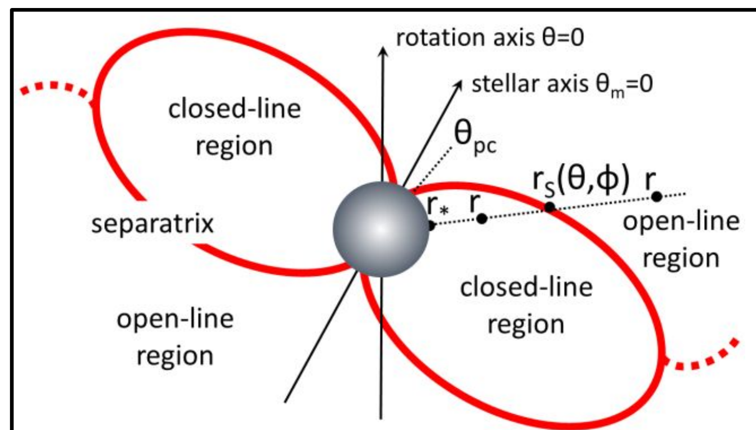
## Novel approach and Neural Networks as solvers




We have decompose the computational domain into two regions, one for the closed and one for the open field lines. Closed line region: Between the central star r = r* and the separatrix r = rS (θ , φ) (thick red line). Open line region: Outside the separatrix (r > rS (θ , φ)).

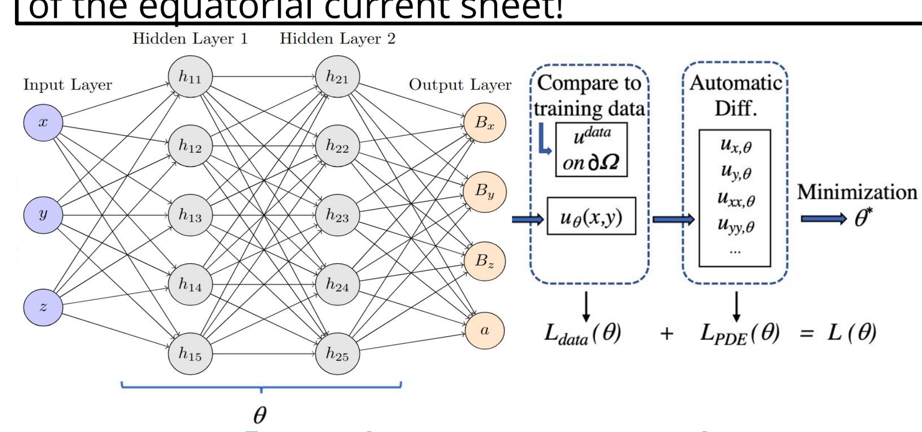
The surface that separates the two regions is the **separatrix surface**, which, in our case, acts as a boundary between the regions with zero thickness. The initial shape of the separatrix is arbitrary; however, to obtain a physically accepted solution, it is essential to determine its final shape in accordance with the physical condition (Lyubarskii (1990)):

$$(B^2 - E^2)_{open} = (B^2 - E^2)_{closed}$$

**We adjust separatrix surface until achieve pressure balance by this recipe:**

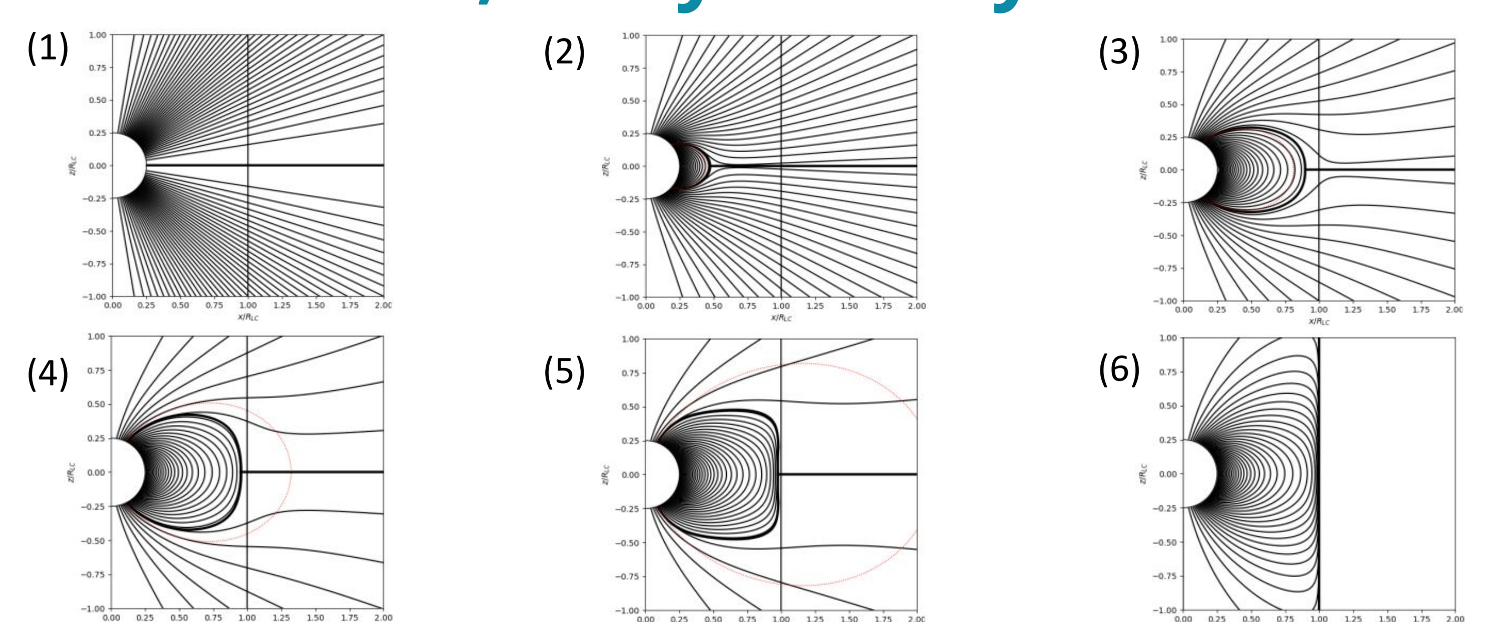
$$r_{S\,\text{new}} = r_S + 2\beta \left(\frac{r_S - r_*}{1 - r_*}\right)^2 \frac{(B^2-E^2)_{r=r_S^-} - (B^2-E^2)_{r=r_S^+}}{(B^2-E^2)_{r=r_S^-} + (B^2-E^2)_{r=r_S^+}}$$

We have implemented one more trick that greatly simplifies our problem, namely a novel numerical treatment of the equatorial current sheet that originates at the tip of theclosed line region. We artificially invert the direction of the field lines that leave the star from the southern pole. This configuration is clearly artificial (it is equivalent to a magnetic monopole), but it is mathematically and dynamically equivalent to the configuration that we are investigating in the open line region, only without the mathematical discontinuity of the equatorial current sheet!




In order to find the solution in the two separated region we used Physics Informed Neural Networks (PINNs). The two NNs has common boundary of the separatrix surface.

## Results in 2D, axisymmetry

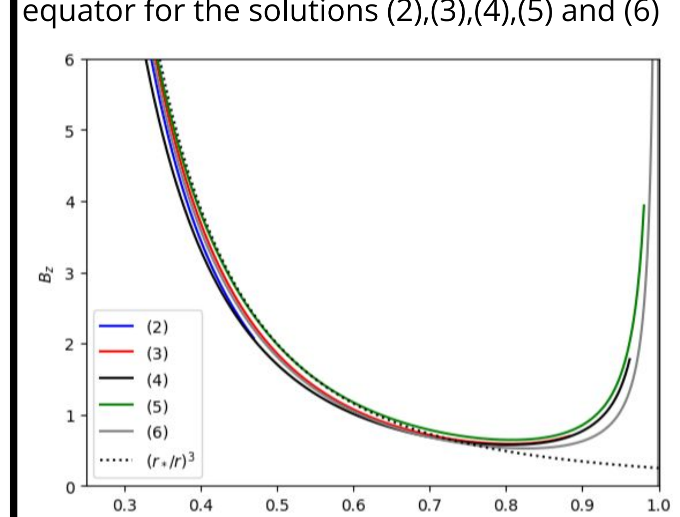



Intermediate solutions for $\theta_{pc} = \pi/2$ and $\theta_{pc} = 2/ 1.176/ 0.9/ 0.7/ 0 \times (r^* /R_{LC})^{(1/2)}$ (from left to right respectively), or equivalently $\Psi_{open} = \Psi_{max}$ and $\Psi_{open} \equiv \Psi_s = \Psi_{max} \sin^2\theta_{pc} = 2.256/1.228/0.756/0.468/0 \times \Psi_{dipole\,LC}$ respectively.

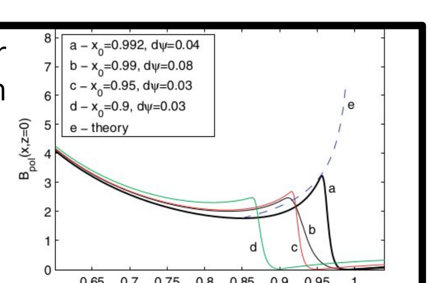
**What we know from the theory is:** $B_p^2(1-x^2)\big|_{IN} = B_p^2(1-x^2)\big|_{OUT} + \frac{I(\Psi_S)^2}{x^2}$

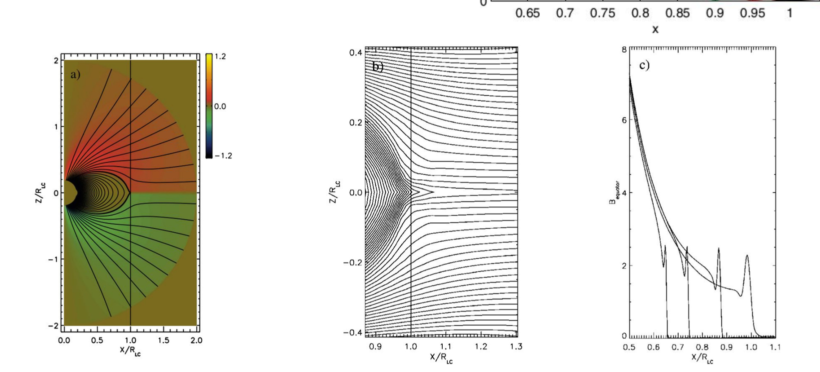
**At the Y-point ( *x* = *x*$_Y$):** $B_p(x_Y)\big|_{IN} = \frac{I(\Psi_S)}{x_Y\sqrt{1-x_Y^2}} = \frac{I(\Psi_S)}{\sqrt{2}\sqrt{1-x_Y}} \to \infty$ when $x_Y \to 1$

Our result for the poloidal field along the equator for the solutions (2),(3),(4),(5) and (6)

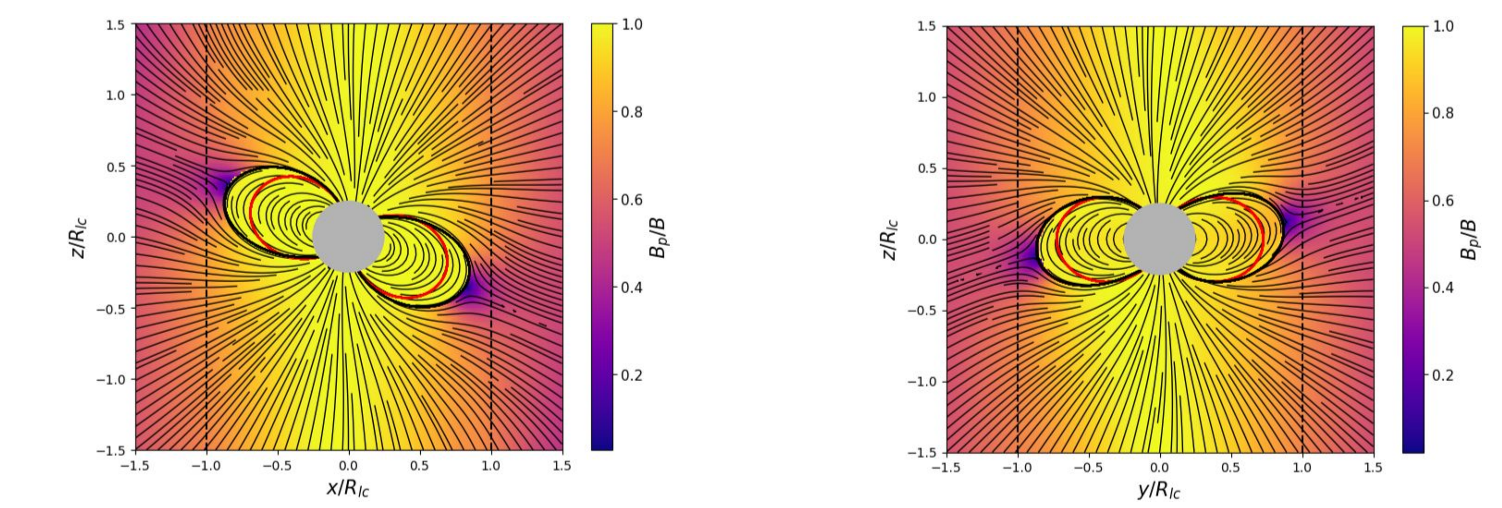


Compare with results from other numerical approaches (Timokhin (2006) and Spitkovsky (2006))

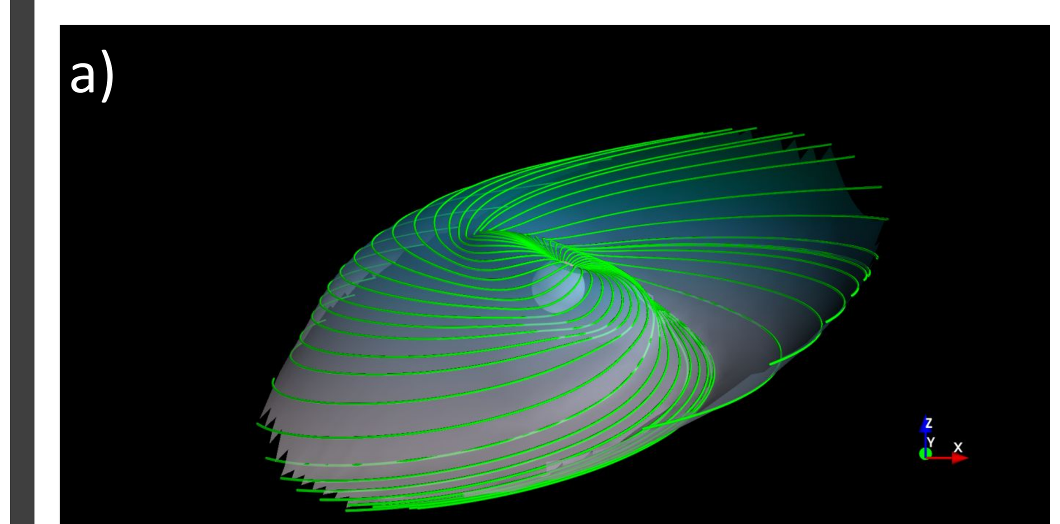


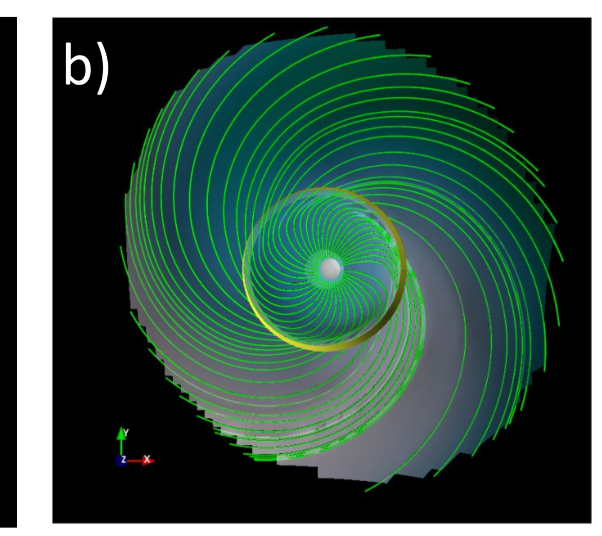



## First Results in 3D, oblique rotator (20°)

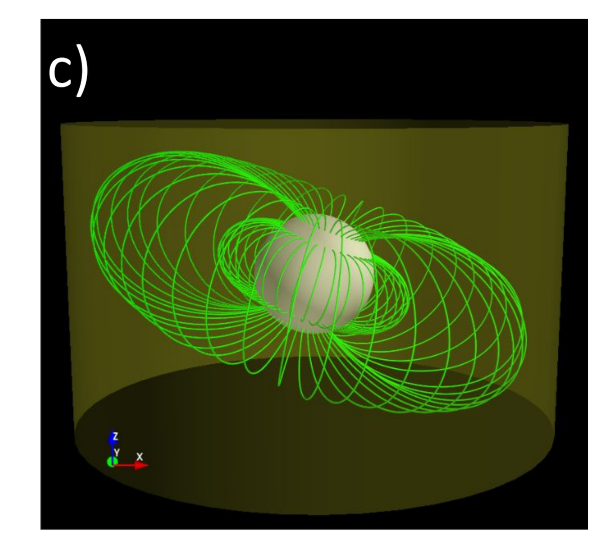


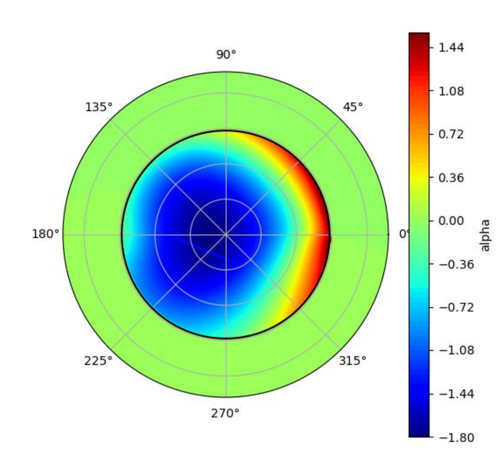
Cross section of steady-state solution representing an inclined rotator with λ = 20° and θpc = 36°. Rotation axis along z. The inclined magnetic axis lies along the corotating xz plane shown. Thick black lines: separatrix between open and closed field lines. Red lines: the initial dipolar shape of the separatrix before readjustment. For this particular choice of the polar cap, the dipole is significantly stretched outwards closer to the light cylinder (represented by the two dashed lines at x/Rlc = ±1). Color scale: ratio $B_\varphi/B$. This represents the development of the azimuthal magnetic field $B_\varphi$ accross the magnetosphere. Notice that at the magnetospheric Y-point where the equatorial current sheet connects to the separatrix current sheet, $B_p = 0$ and $B_\varphi \neq 0$ as expected (Uzdensky 2003)



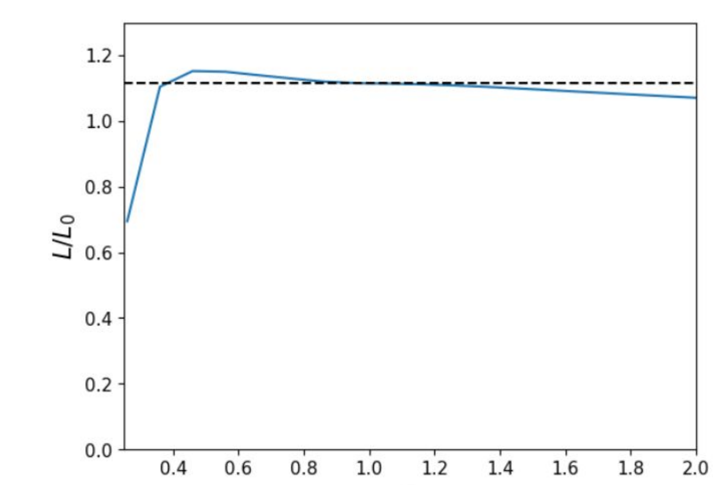




The steady-state solution in 3d plots. In plot a) and b) we plot only open magnetic field lines from the rim of the northern polar cap. In plot a) the undulating shape of the equatorial current sheet is clearly seen. In plot b) we see the clear azimuthal break of open field lines expected very close to the Y-point where $B_p \to 0$ and $B_\varphi \neq 0$. Transparent yellow surface: light cylinder. In plot c) we have a close-up view near the stellar surface in the closed line region. In this solution we required that α = 0 in that region. Nevertheless, we observe that field lines develop a clear azimuthal twist with respect to the magnetic axis.



Distribution of current parameter α along the stellar surface as seen from above the axis of rotation. x/y axes along ϕ = 0◦ /90◦ respectively. α = 0 in the green closed-line region outside the polar cap. Blue region:main magnetospheric current. Yellow-red region: part of the return current



Evolution with distance of the total Poynting flux L calculated over spheres centered over the central star, normalized with respect to the aligned rotator's canonical luminosity value $L_0$ ≡ B2∗ r∗6 c/(4R4lc ). The value of L agrees with previous estimates in the literature (dashed line). Energy is almost conserved beyond the Y-point and the light cylinder due to the absence of dissipation in the magnetospheric current sheets.

### Future Work Plan

We plan to apply our methodology to other oblique angles of the magnetic axis, as well as to different opening angles of the polar cap, aiming to obtain solutions similar to the axisymmetric case. Additionally, another interesting application would be to explore different polar cap shapes, studying solutions that could be compared with observations from the NICER telescope.