Non-linear Steepening of Fast Magnetosonic Waves and Monster Shocks

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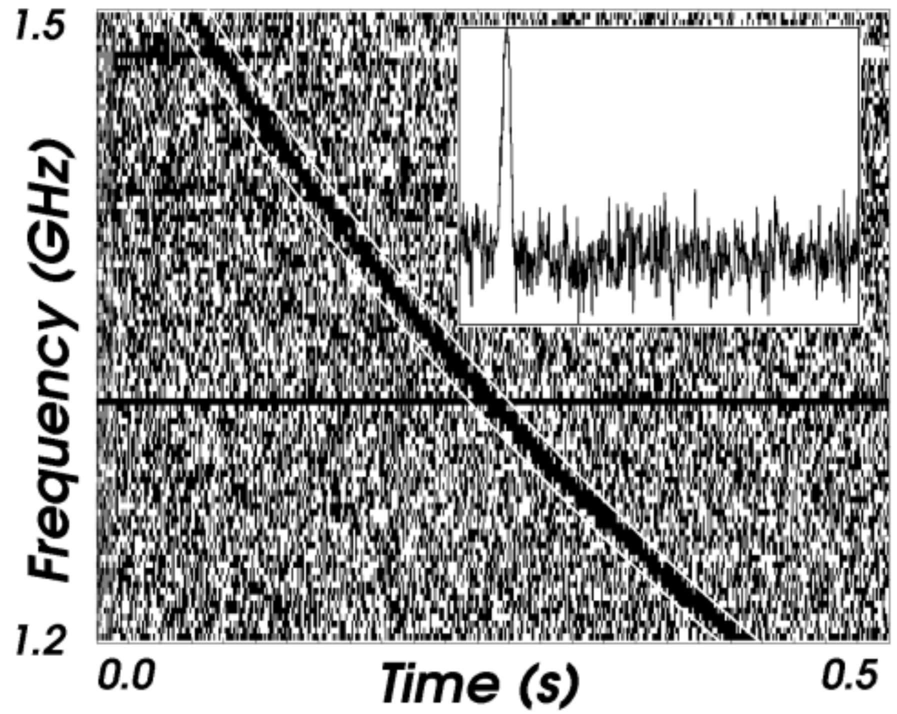
Fast Radio Bursts

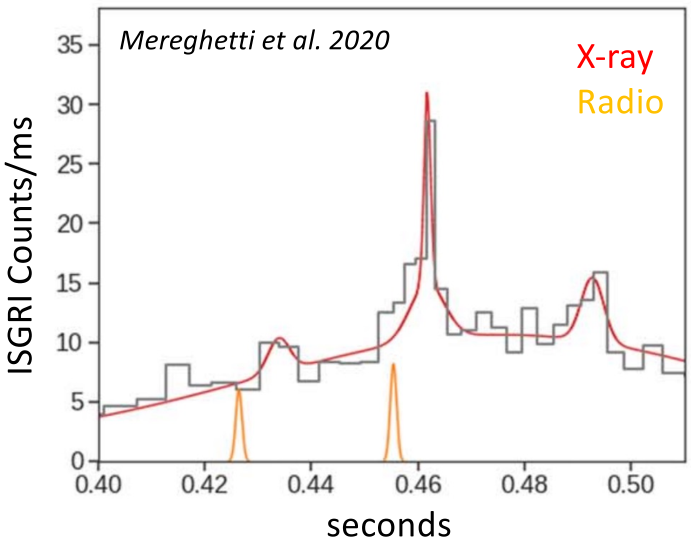
- Fast Radio Bursts (FRB)s are millisecond duration, highly luminous, radio transients
- They appear isotropic in the sky
- They often have extragalactic origin
- Some repeat, some have not
- Many models favor emission from a magnetar



Lorimer et. al 2007

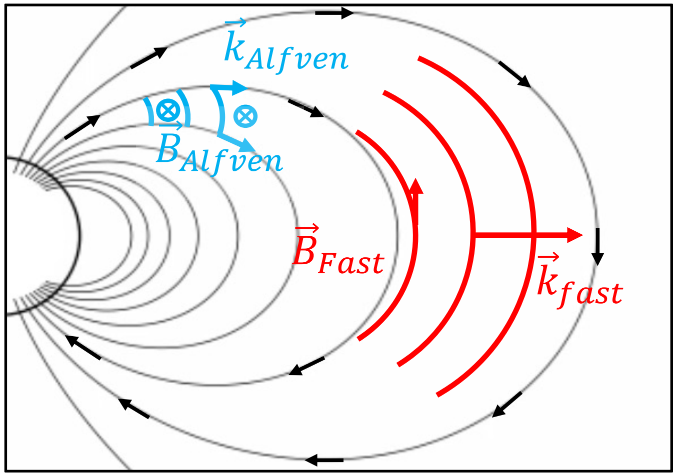
FRB 200428

- FRB 200428 originated from a galactic magnetar (SGR 1935+2154)
- Associated with an unusual X-ray burst
  - Hardest X-ray spectrum ever produced from this magnetar (peaked at ~84 keV)
  - Brighter than most bursts
  - Non-thermal spectrum, unlike typical bursts from this magnetar
  - Two simultaneous peaks in the X-ray and radio light curves



Starquakes and plasma waves

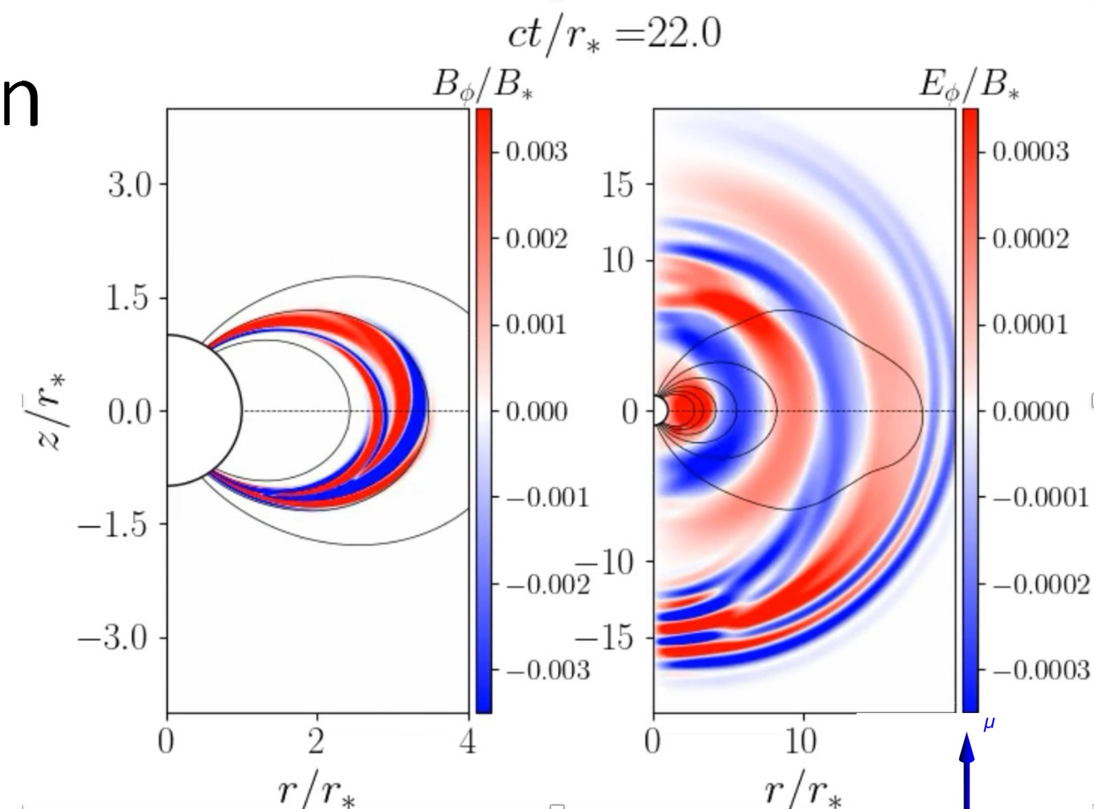
- A simple model for the triggering of transients is shearing of magnetic foot-points of stars
- This can launch Alfvén waves and fast magnetosonic (fast) waves
- Alfvén waves propagate along magnetic field lines
  - Alfvén waves in a dipolar magnetosphere spontaneously convert to fast waves

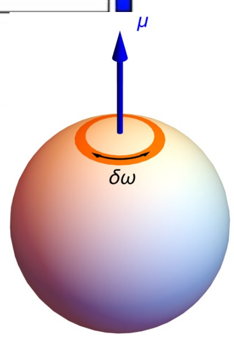


Alfvén wave conversion

Bernardi et. al 2025

- Alfvén waves spontaneously convert into fast waves
- The structure of the fast wave is determined by two things
  - High frequency front: twice the Alfvén wave frequency
  - Low frequency mode: bouncing time of the Alfvén wave train
- Low frequency mode is more robust

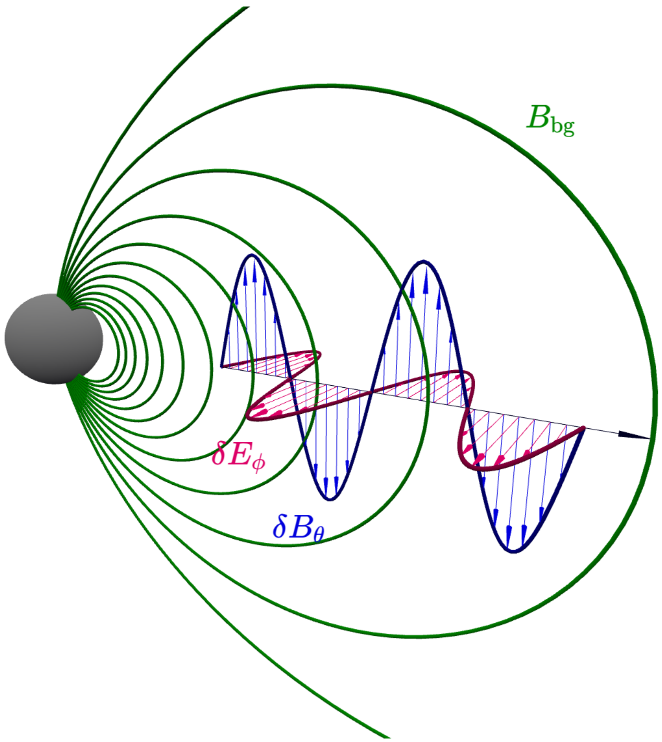




<u>Magnetars are dynamic and will inevitably produce energetic fast magnetosonic waves</u>

Fast Magnetosonic waves

- Restoring Force: Magnetic Pressure
- Free to propagate across magnetic field lines
- Magnetic field in the $\vec{k} - \vec{B}$ plane
- In the infinite magnetization limit they become electromagnetic waves
  - Reminder, magnetization $\sigma = \frac{B^2}{4\pi\rho c^2\gamma}$
- Spherical waves fall off as $\frac{1}{r}$
- Magnetic dipoles fall off as $\frac{1}{r^3}$

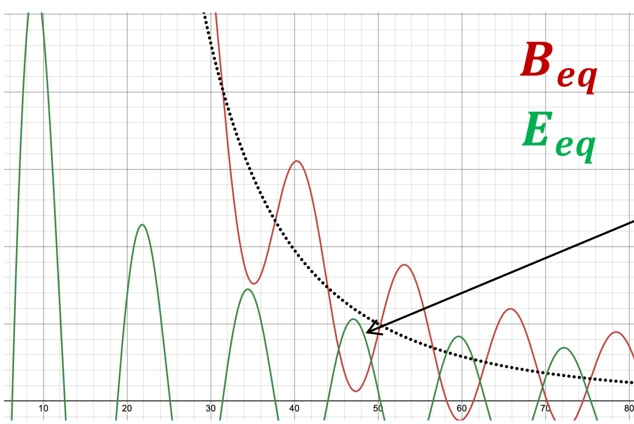


Vacuum Waves in Equatorial Plane

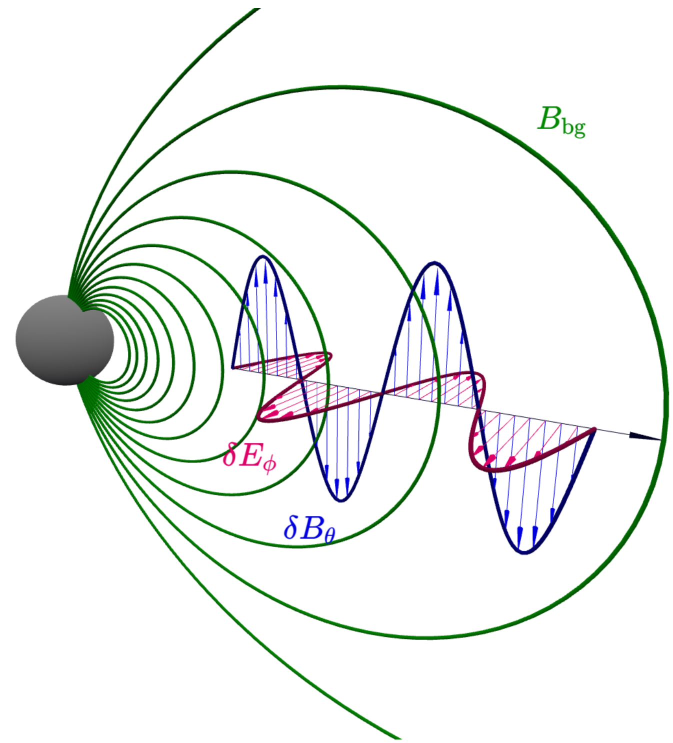
Equatorial fields if the wave doesn't deform

$$\boldsymbol{B_{eq}} = \frac{B_*}{r^3}\widehat{\boldsymbol{\theta}} + \frac{\delta B}{r}\sin(kr - \omega t)\,\widehat{\boldsymbol{\theta}}$$

$$\boldsymbol{E_{eq}} = \frac{\delta B}{r}\sin(kr - \omega t)\,\widehat{\boldsymbol{\phi}}$$



This is not allowed in a dense plasma



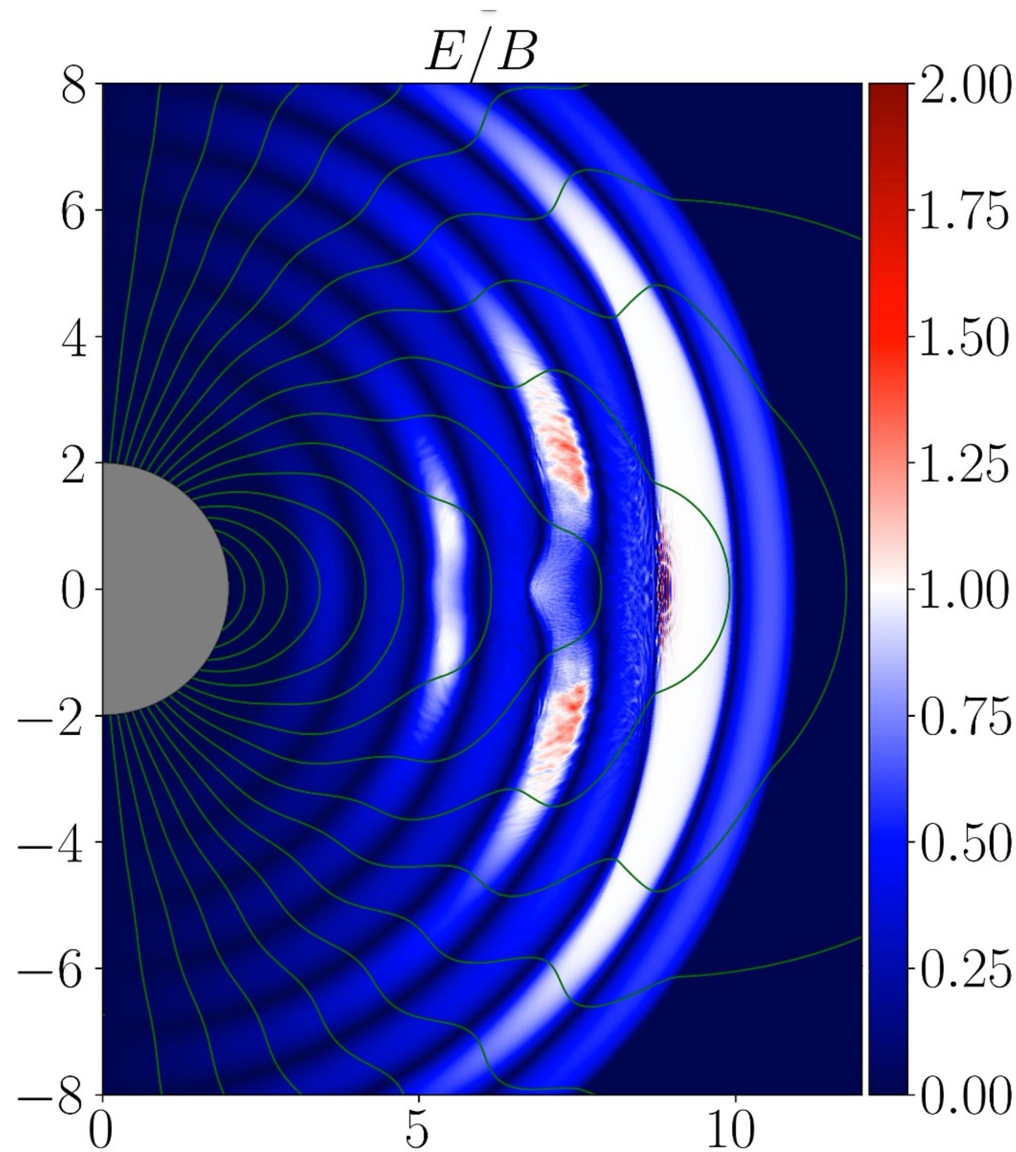
Introducing Monster Shocks

- In a sufficiently dense plasma, the waves behave according to MHD
  - $E \to B$
  - Wave deforms leaving $E \approx B$
  - Drift velocity $\frac{E \times B}{B^2} c \to c$
    - In the equatorial plane, this is directed radially inwards
  - All the energy removed from the wave gets dumped into the plasma
    - Upstream plasma develops a Lorentz factor proportional to the background magnetization
    - $\gamma \propto \sigma_{bg}$
  - Wave fields change signs $\Longrightarrow$ drift velocity changes signs
  - A shock forms

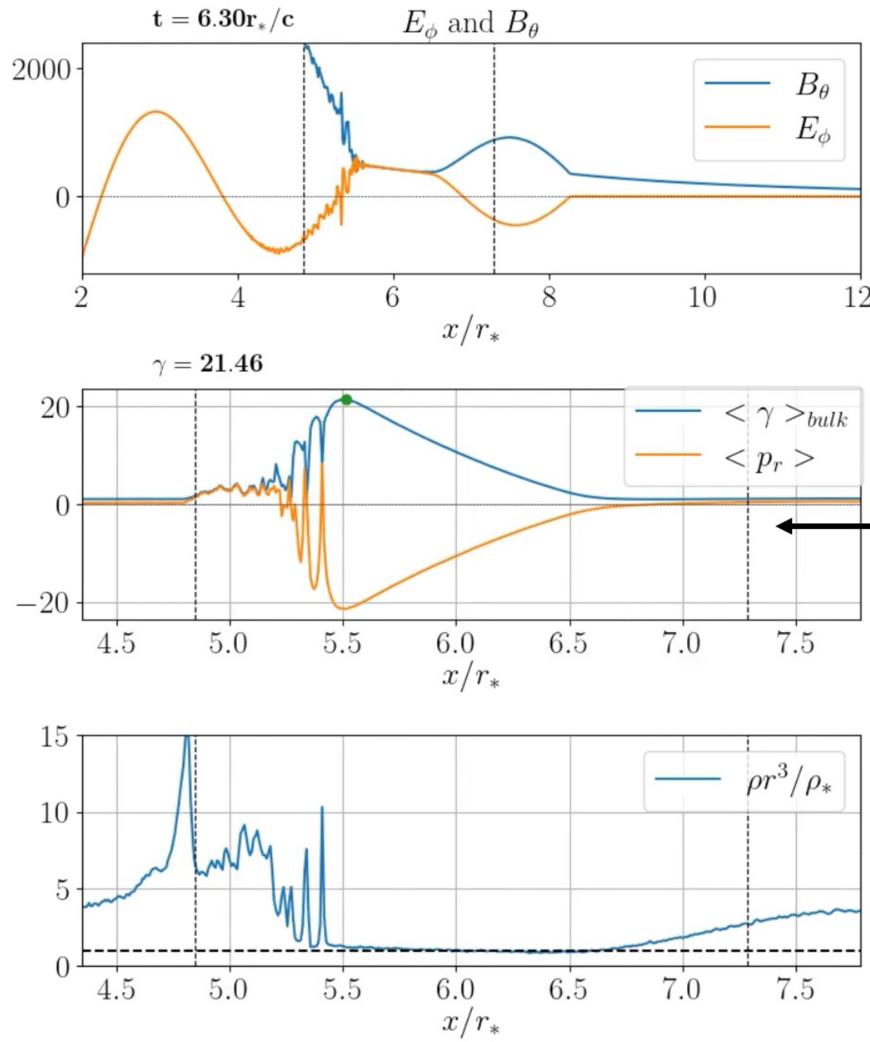
Chen et. al. 2022, Beloborodov 2022, Vanthieghem et. al 2024

Particle In Cell

- Density and magnetic field $\sim \frac{1}{r^3}$
- Wave profile $\sim \sin(\theta)$
- Directly impose electric field at boundary
- Scale separation varies from simulation to simulation
  - Typical: $\lambda_{fast} \gtrsim 100\lambda_{plasma}$ at nonlinear radius



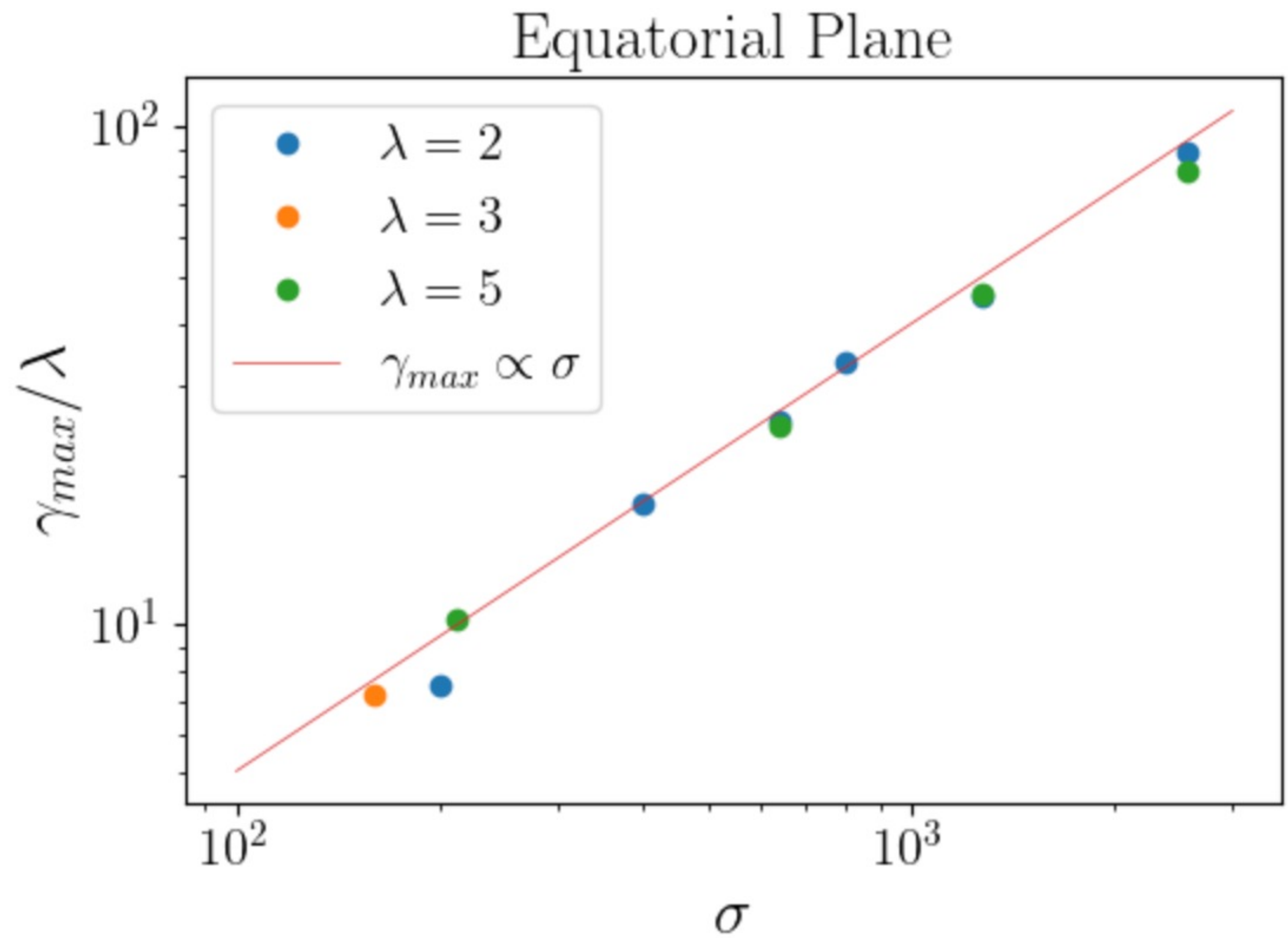
$$\sigma_{\mathrm{bg,nl}} = 160$$



$$E \perp B$$

$$|E| \approx |B|$$

$$\frac{E \times B}{B^2} c$$



Equatorial Plane
10^2
10^1
$\lambda = 2$
$\lambda = 3$
$\lambda = 5$
$\gamma_{max} \propto \sigma$
γ_{max}/λ
10^2
10^3
σ

- Characteristic frequency of magnetar quakes

$$\omega = 10^5 \omega_5 rad/s$$

- Luminosity of X-ray burst associated with FRB 200428

$$L = 10^{40} L_{40} erg/s$$

- Surface magnetic field strength of SGR 1935+2154

$$B = 10^{14} B_{14} G$$

- Fast wave with this luminosity will steepen at

$$r_{nl} \sim 3\times10^2 r_* B_{14}^{1/2} L_{40}^{-1/4}$$

- Local magnetization given a multiplicity of $\mathcal{M} = 10^6 \mathcal{M}_6$

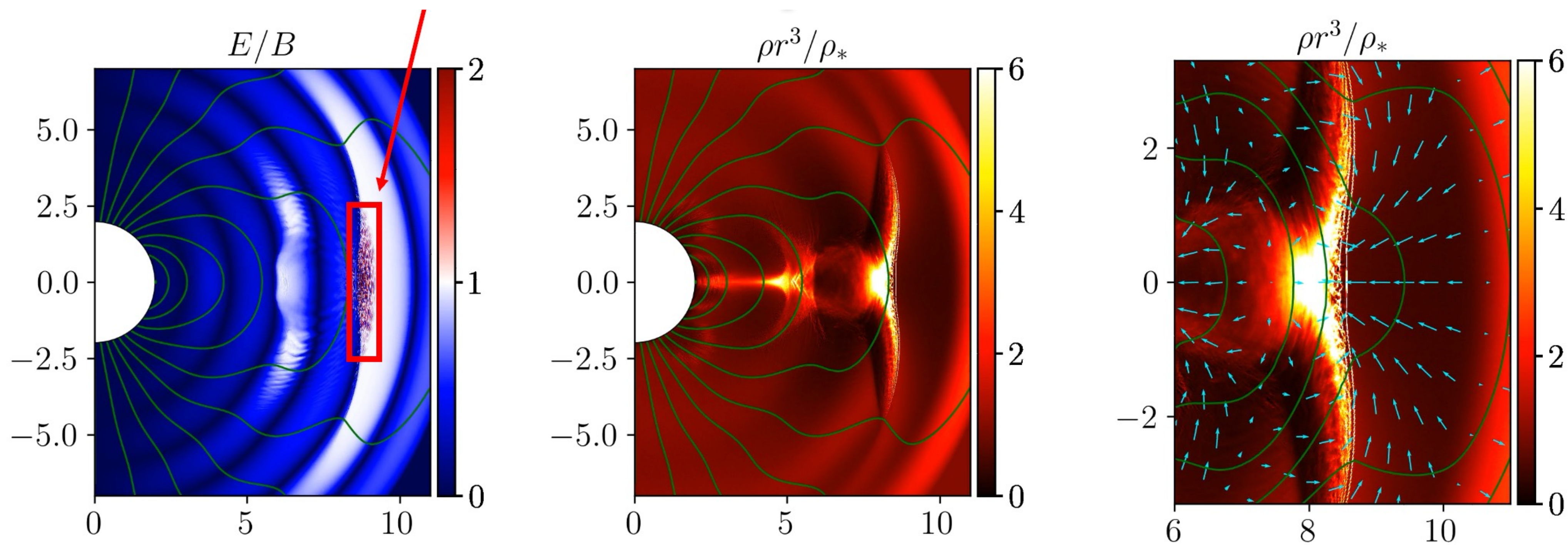
$$\sigma_{nl} \sim 5\times10^6 B_{14}^{-1/2} L_{40}^{3/4} \mathcal{M}_6^{-1}$$

- Bulk Lorentz factor

$$\gamma_{up} \sim \frac{c\sigma_{nl}}{\omega r_{nl}} \sim 5\times10^3 B_{14}^{-1} L_{40} M_6^{-1} \omega_5^{-1}$$

- Synchrotron spectrum would peak at

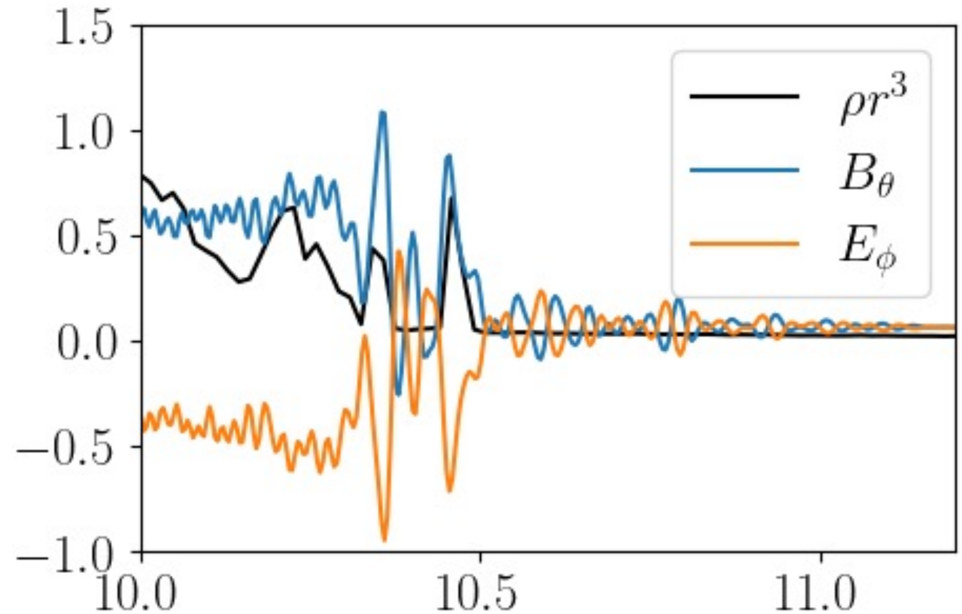
$$2\ B_{14}^{-5/2} L_{40}^{11/4} \mathcal{M}_6^{-2} \omega_5^{-2} MeV$$



E/B
$\rho r^3/\rho_*$
$\rho r^3/\rho_*$

Precursor wave

- Unique feature of perpendicular magnetized shocks
- Plasma moving across shock front hits a jump in the magnetic field
- Gyrations form a resonance cavity
- Excites a precursor wave that propagates upstream of the shock



Plotnikov et. al 2019, Vanthieghem et. al 2024

- We had:

$$L = 10^{40} L_{40} erg/s,\ \omega = 10^5 \omega_5 rad/s,\ \ B = 10^{14} B_{14} G,\ \mathcal{M} = 10^6 \mathcal{M}_6$$

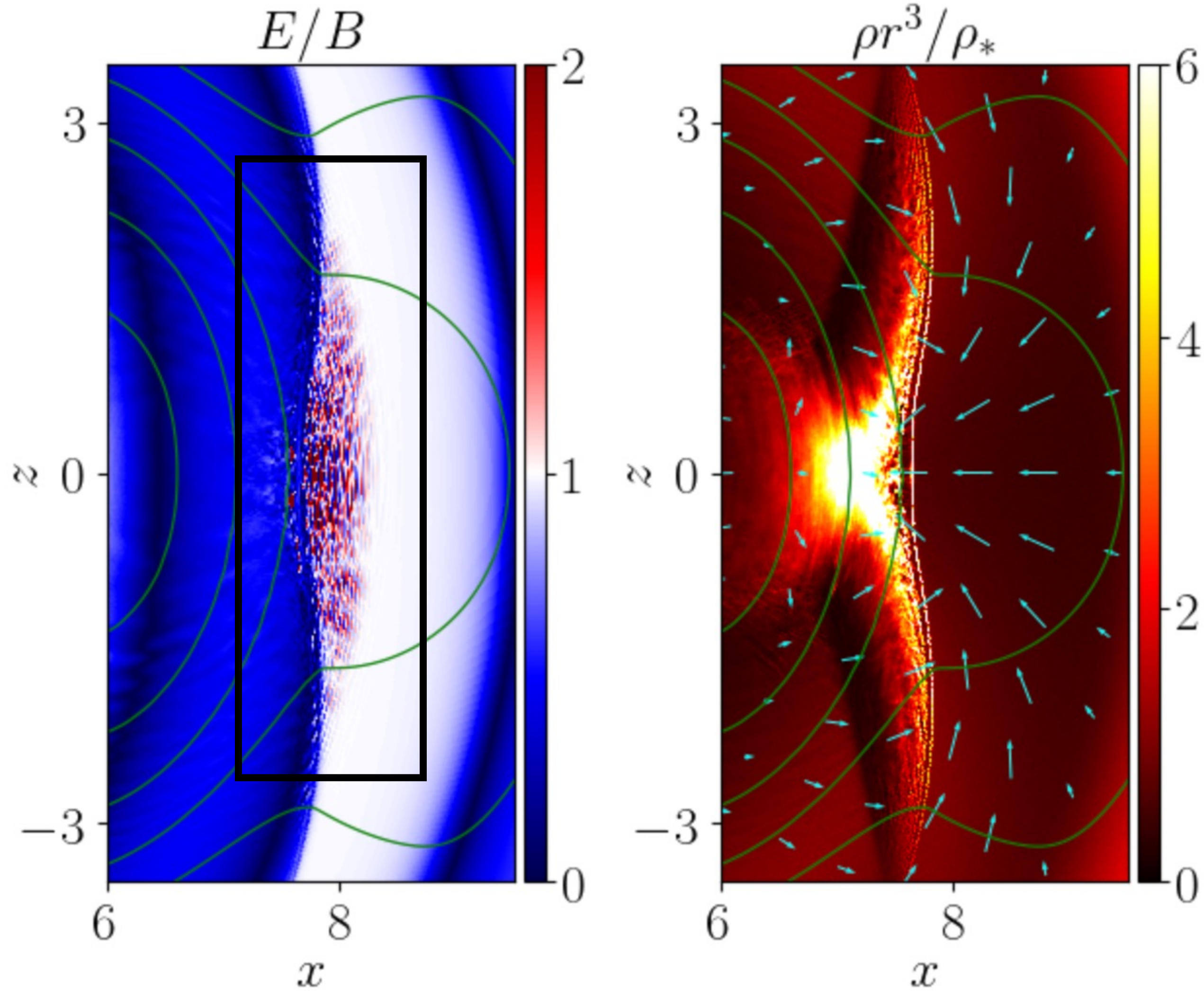
- The precursor wave frequency in the lab frame will be peaked at

$$\omega_{peak} = \gamma_{sh} \omega'_p$$

ω'_p is the proper plasma frequency in the upstream and $\gamma_{sh} \sim \sqrt{\sigma_{up}}$

- Which for this burst

$$\omega_{peak} = 6\, B_{14}^{1/2} L_{40}^{-1/4} \mathcal{M}_6 \omega_5 GHz$$



E/B
$\rho r^3/\rho_*$
3
0
−3
6
8
x
z
2
1
0
6
4
2
0

Conclusions

- Using Particle In Cell simulations we have directly verified properties of monster shocks
  - The upstream Lorentz factor is proportional to the magnetization and the wavelength of the fast wave
- This could be a source of the non-thermal X-ray bursts observed from SGR 1935+2154
- Monster shocks produce a precursor wave suggestive of fast radio bursts
  - The anisotropy of the precursor emission may explain why magnetar X-ray bursts are rarely associated with FRBs