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Rotational evolution of an (accreting) neutron star and its governing torques

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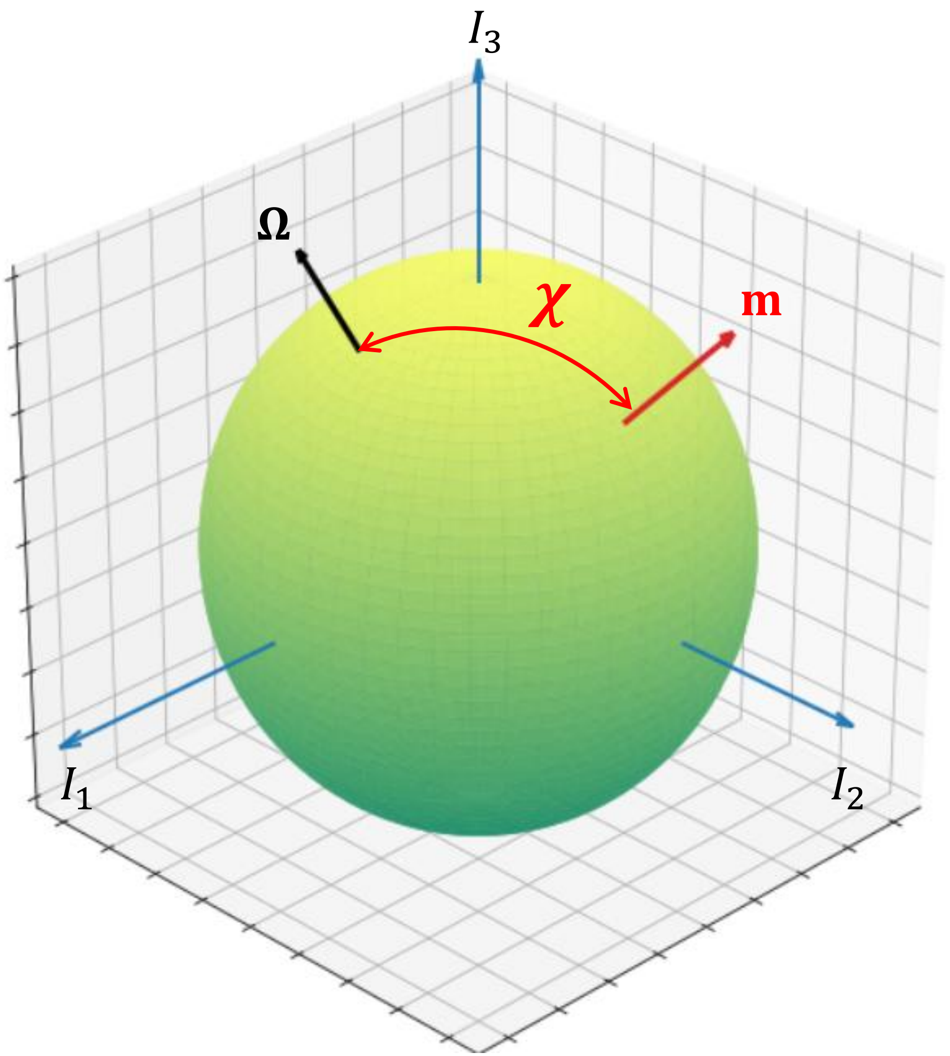
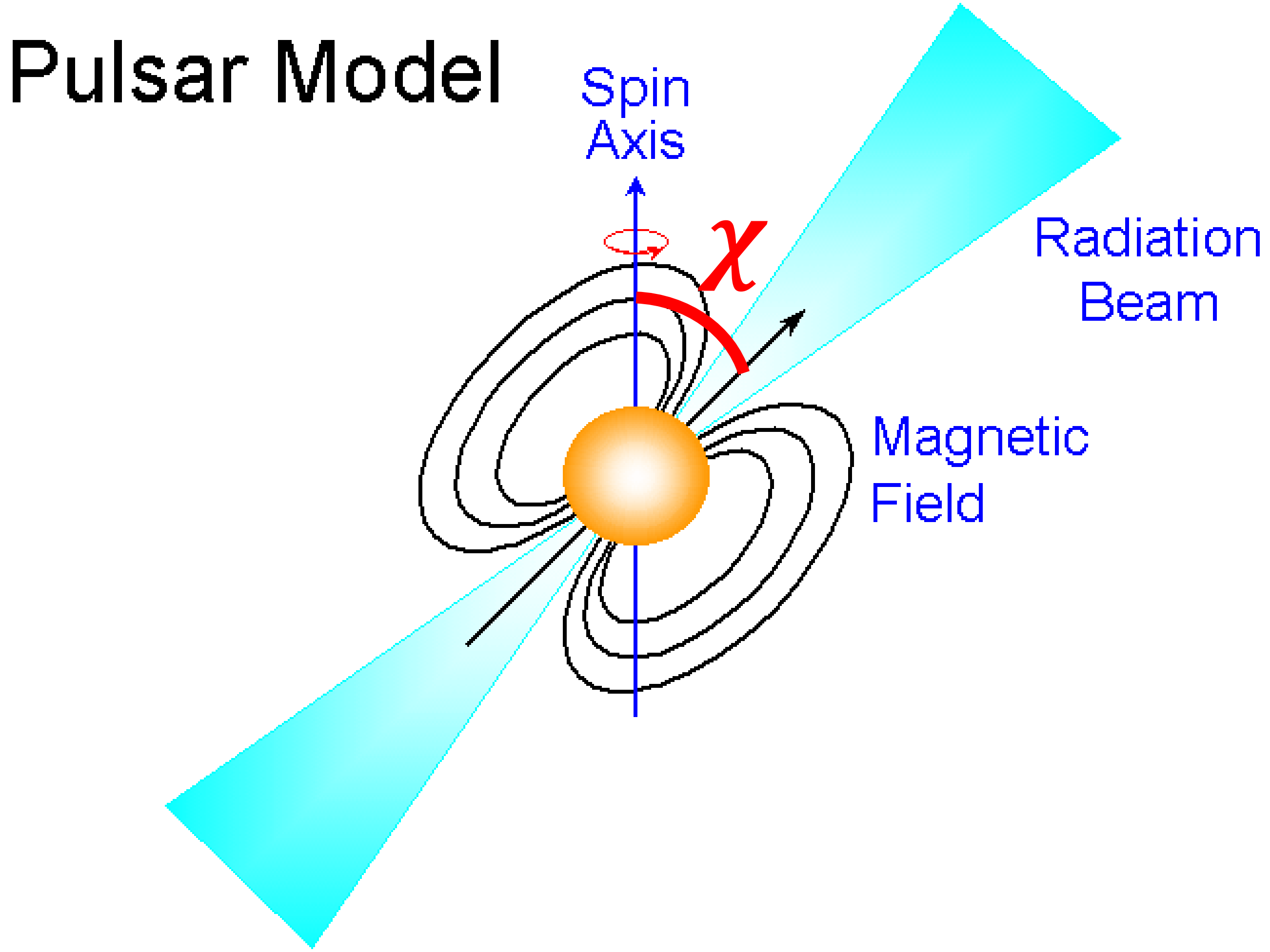
Les Houches - 10 Apr 2025

NS rotational evolution: the magnetic angle

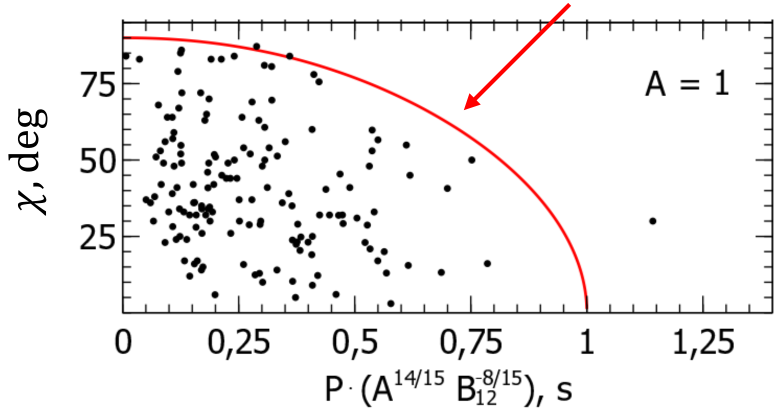




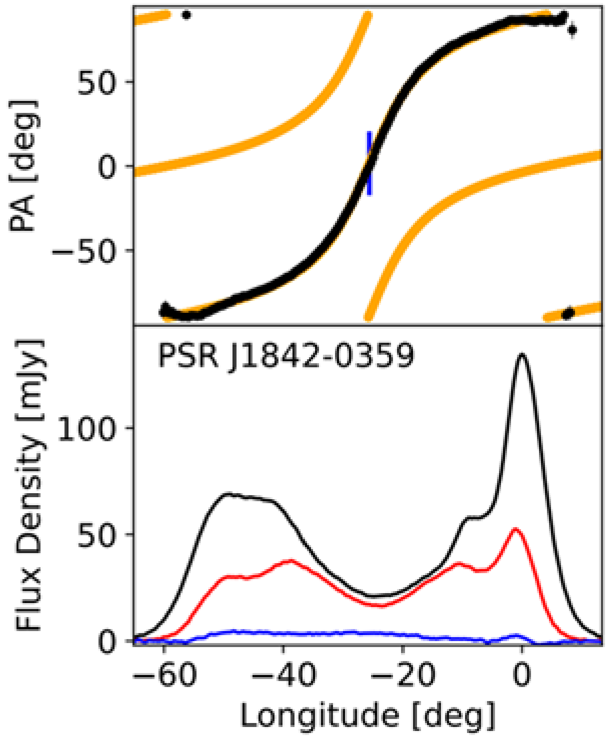
Magnetic angle in NS's everyday life

Magnetic angle is important for:

- Spin-down evolution
- Pulse width and form
- Polarization profile
- Accretion flow structure
- Apparent statistics of neutron stars



Novoselov et al., 2004.03211



Johnston et al., 2212.03988

Evolution of a spherical isolated NS:

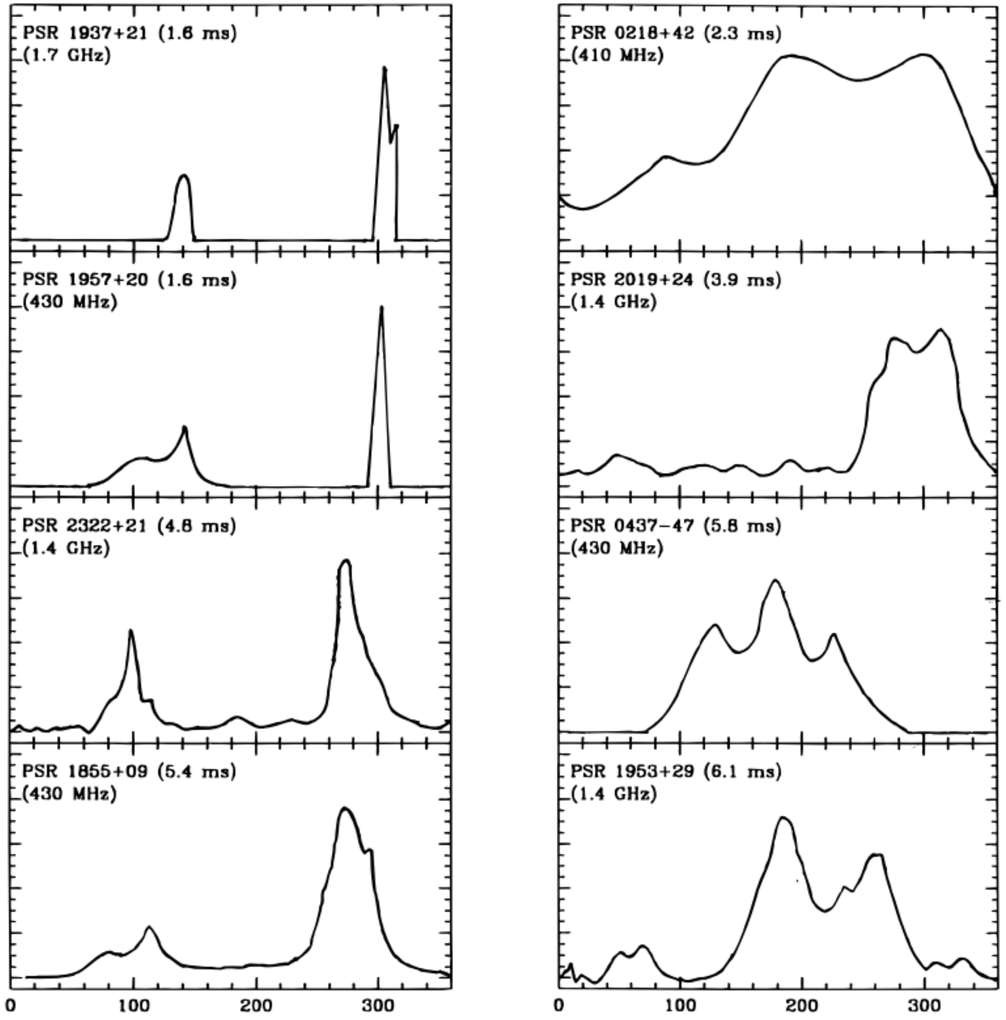
$$\dot{\Omega} \propto -B^2\Omega^3 \cdot (k_0 + k_1 \sin^2\chi)$$

$$\dot{\chi} \propto -k_2 B^2 \Omega^2 \cdot \sin\chi\cos\chi$$

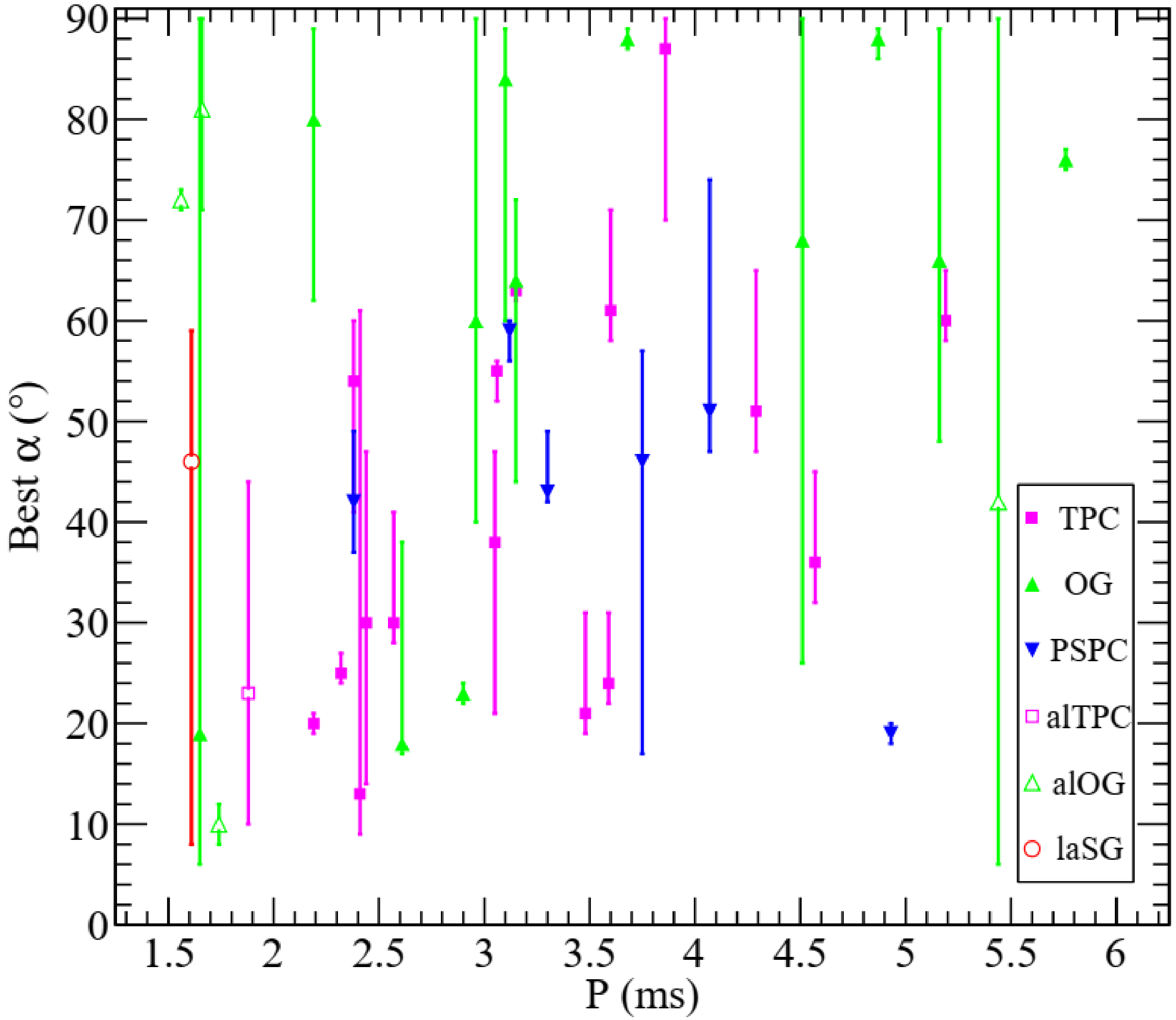
$\tau_{\rm align} \sim 10^8 I_{45} P^2 \mu_{30}^{-2}$ yr

Philippov et al., 1311.1513

Magnetic angles of millisecond pulsars

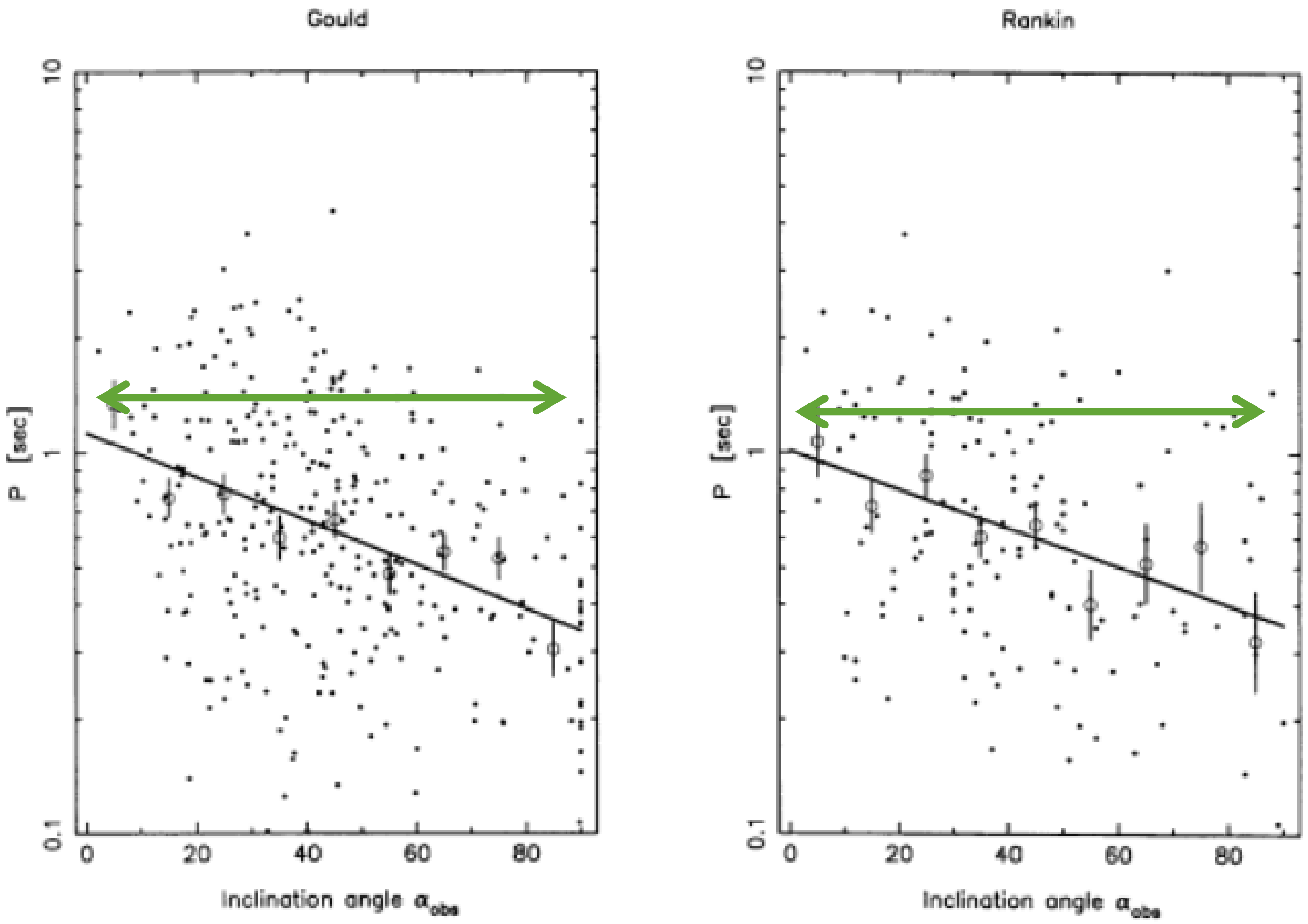


Chen et al., 1998, ApJ, 493, 397

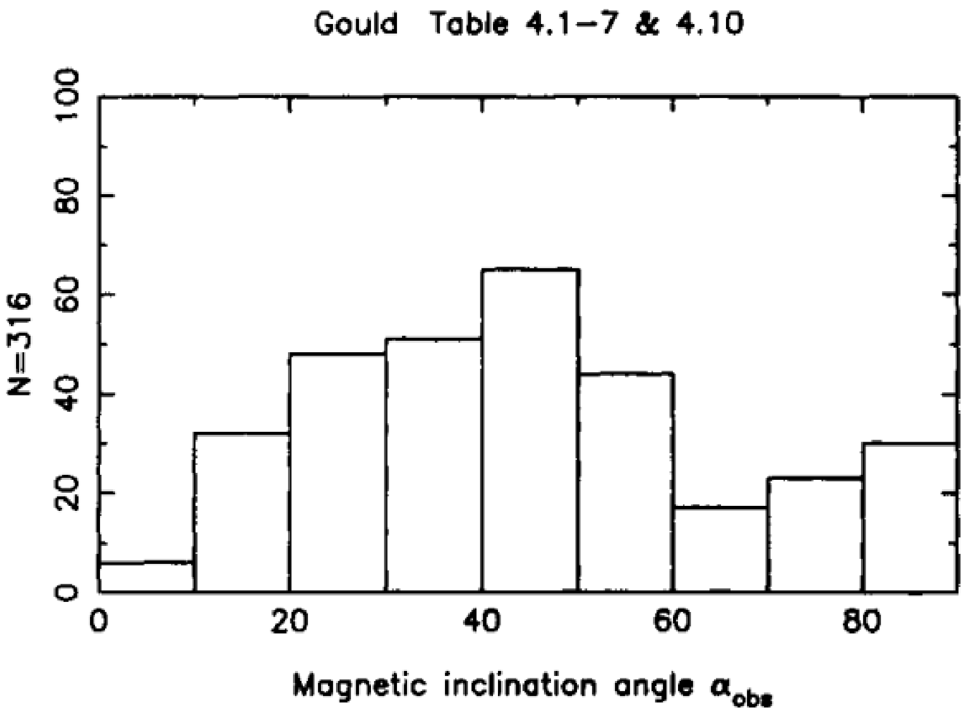


Johnston et al., 1404.2264

Magnetic angles of rotational powered pulsars



Tauris & Manchester, 1998, MNRAS, 298, 625

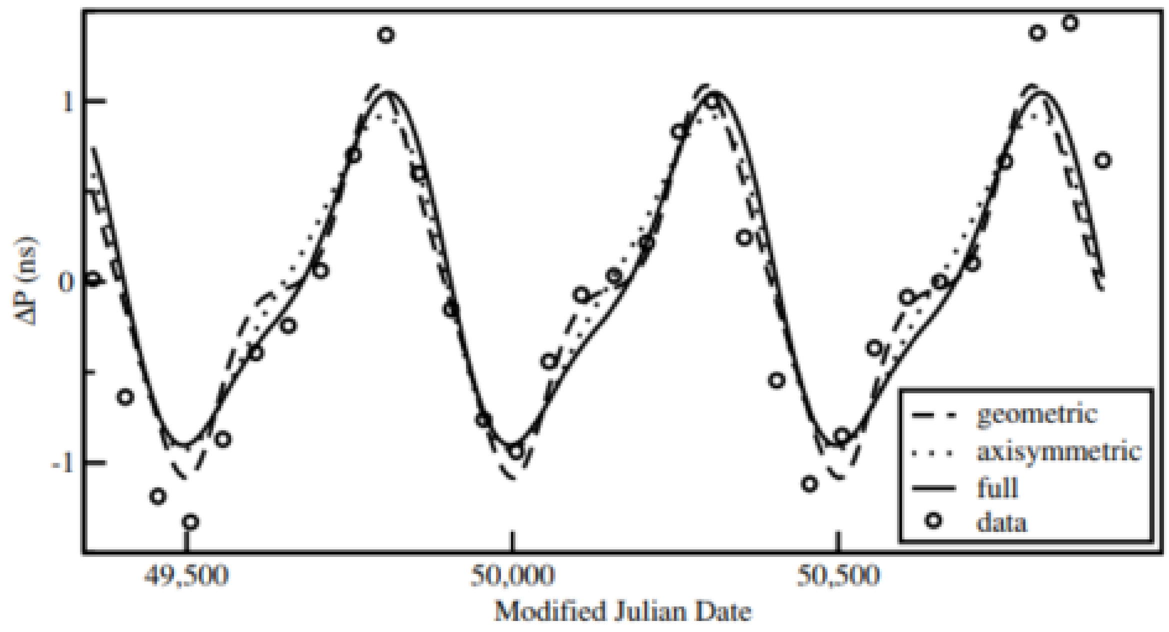


Too many ‘nearly orthogonal’ pulsars?
Magnetic orthogonalization?

Novoselov et al., 2004.03211
Knyazev, Beskin & Istomin, accepted

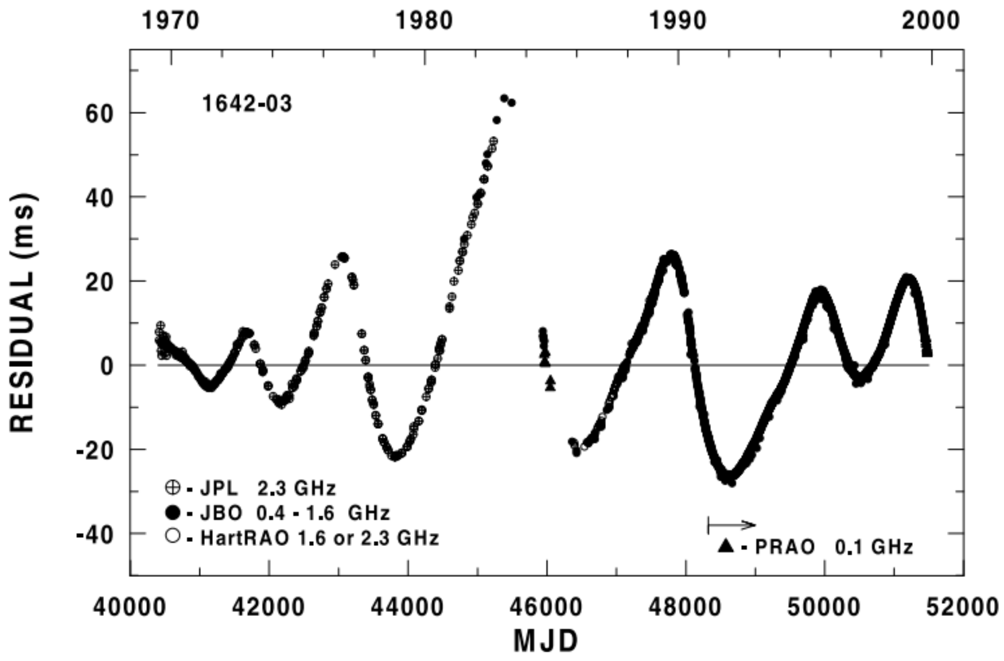
NSs precession: pulsars

PSR B1828-11, $P_{\mathrm{spin}} = 0.405\ \mathrm{s},\ T_{\mathrm{p}} \sim 500\ \mathrm{d}$



Akgun et al., astro-ph/0506606

PSR B1642-03, $P_{\mathrm{spin}} = 0.388\ \mathrm{s},\ T_{\mathrm{p}} \sim 3-7\ \mathrm{yr}$

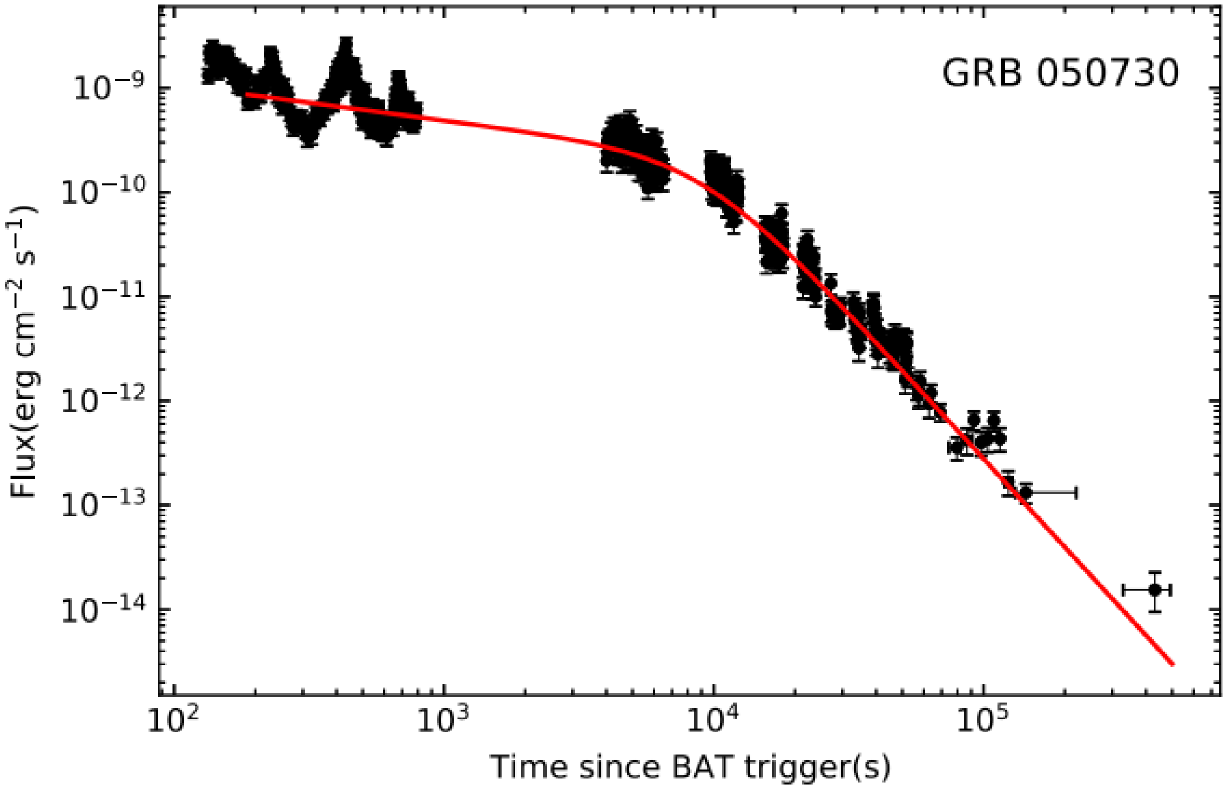


Shabanova et al., astro-ph/0101282

Star deformation: $\varepsilon \sim 10^{-9}..10^{-8}$

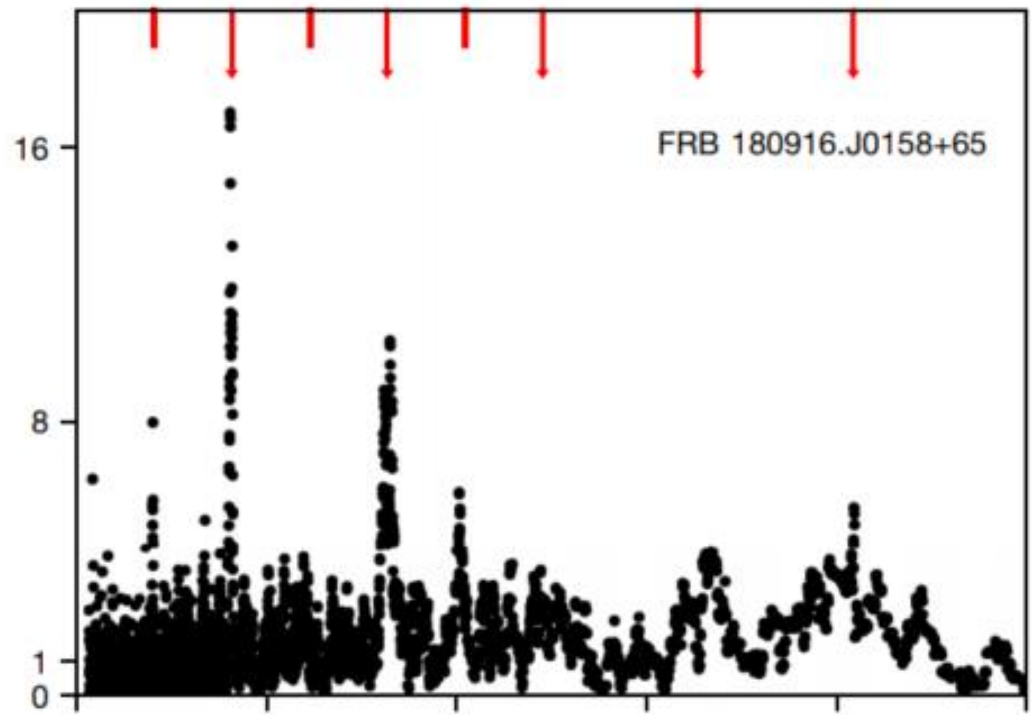
NSs precession: magnetars(?)

GRB 050730 , $T_{\rm QPO} \sim 250\ s,\ \ P_{\rm spin} \sim 4\ ms\ (?\,)$



Zhang et al., 2411.15883

FRB 180916.J0158+65, $T_{\rm QPO} \sim 16.5\ {\rm d}, P_{\rm spin} \sim 2\ {\rm s}\ (?\,)$

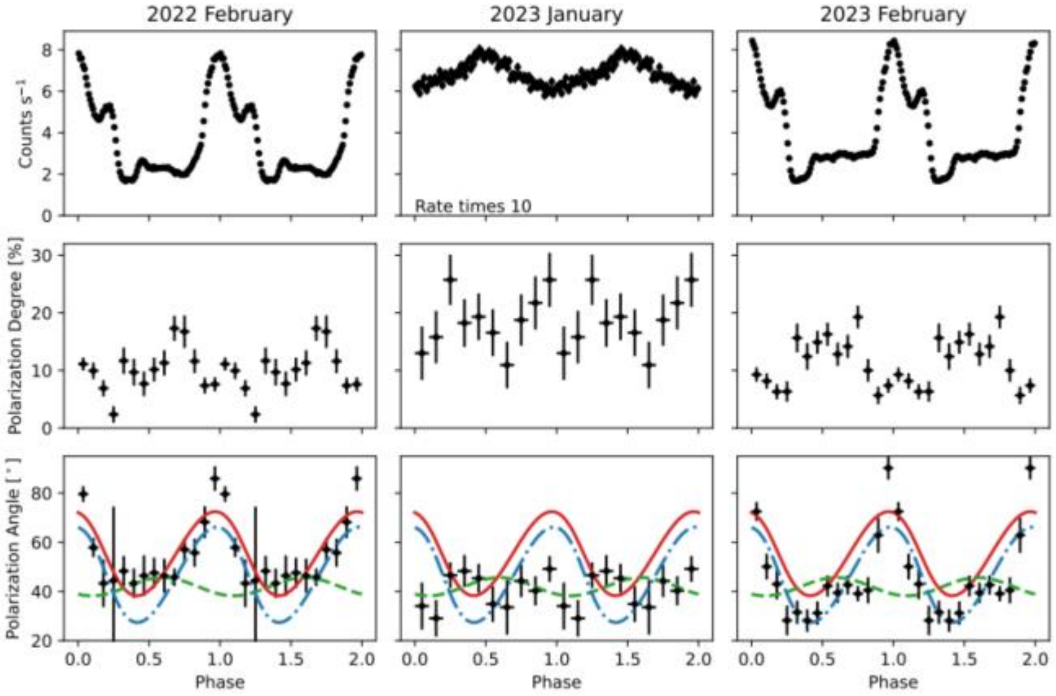


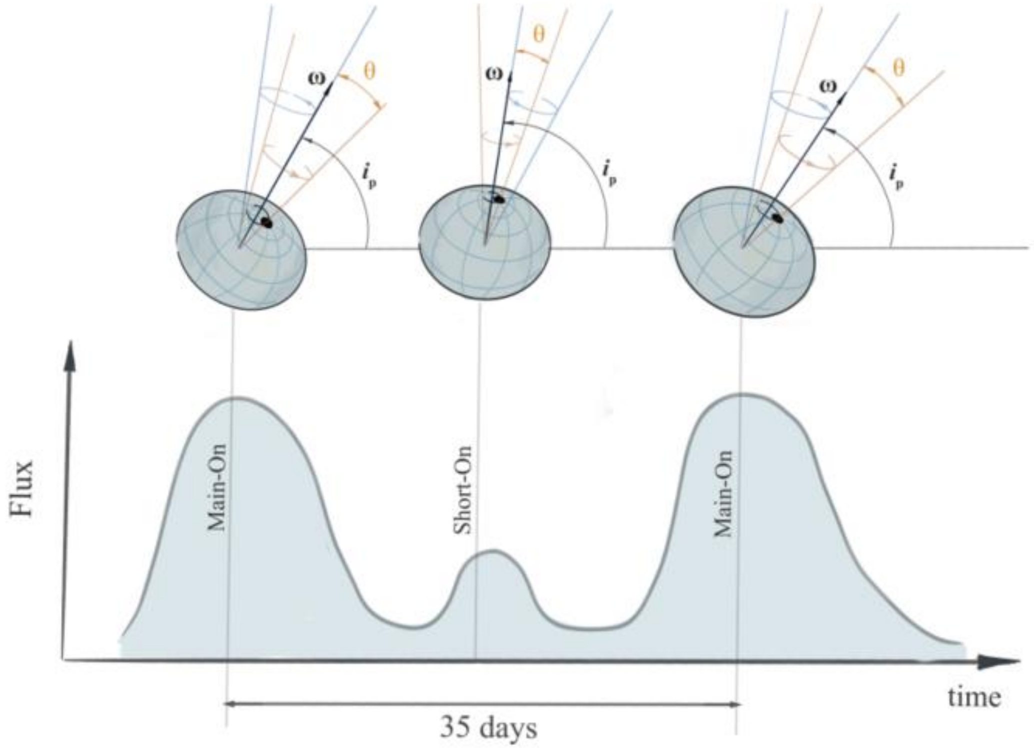
The CHIME/FRB Collab., 2001.10275
See also Levin et al., 2002.04595

$$\varepsilon \sim 10^{-6}..10^{-5}$$

NSs precession: Her X-1

$P_{\rm spin} = 1.24\ {\rm s}, T_{\rm p} = 35\ {\rm d}$



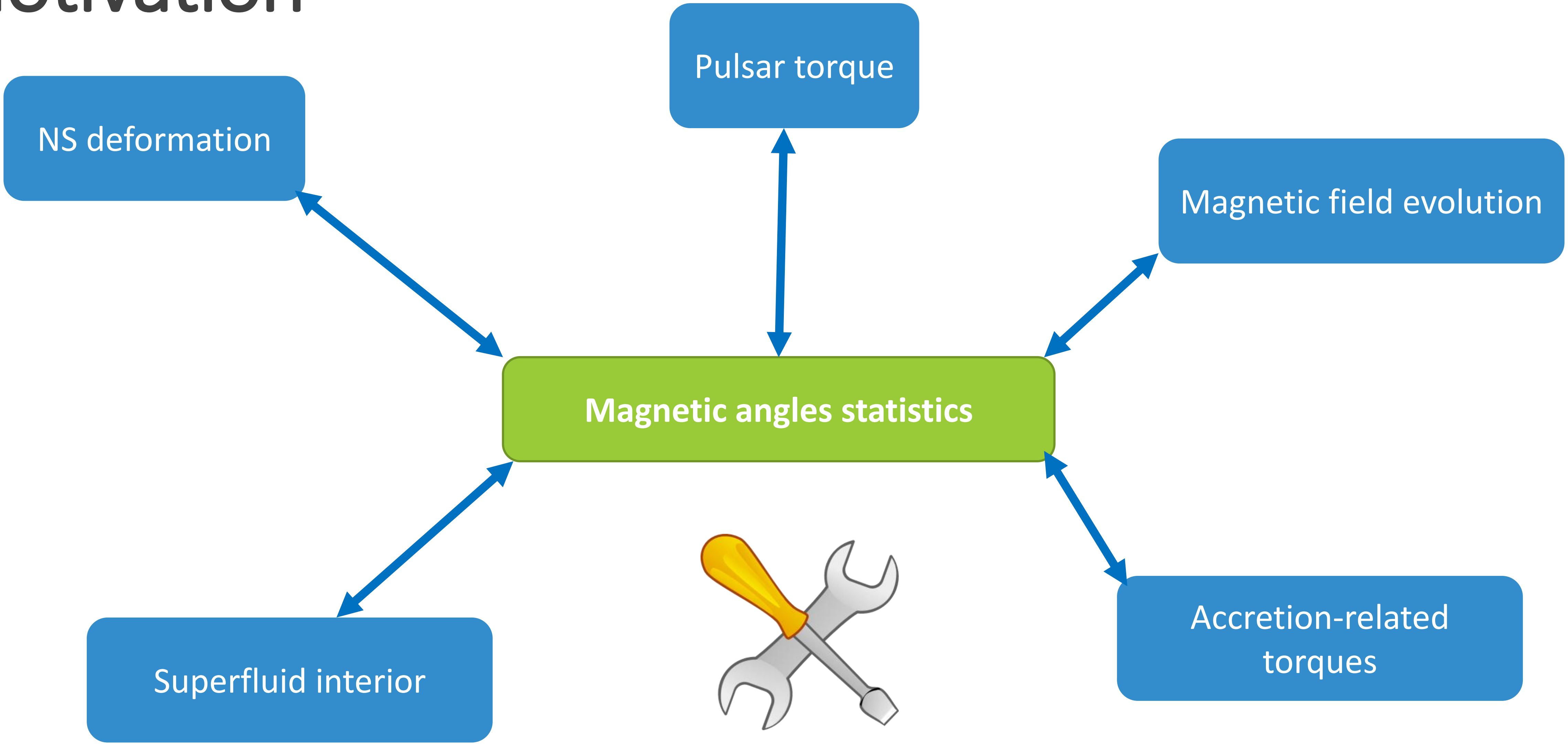


Heyl et al., 2311.03667

See also Kolesnikov et al., 2204.06408

$\varepsilon \sim 4 \cdot 10^{-7}$

Motivation



Pulsar torque
NS deformation
Magnetic field evolution
Magnetic angles statistics
Superfluid interior
Accretion-related torques

A neutron star torques

- Isolated neutron star:

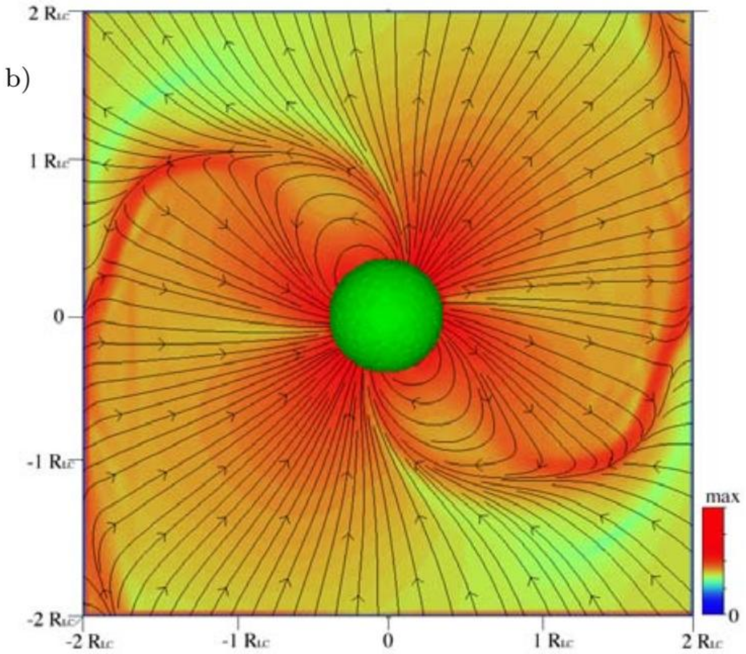
$$\boldsymbol{N} = \boldsymbol{N}_{\mathrm{psr}}$$ - Standard pulsar spin-down

- Accreting neutron star:

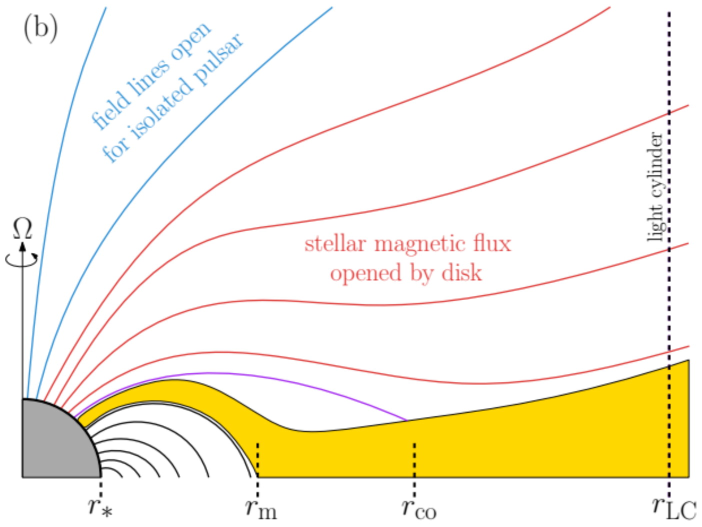
$$\boldsymbol{N} = \left(\frac{r_{\mathrm{LC}}}{r_{\mathrm{m}}}\right)^2 \boldsymbol{N}_{\mathrm{psr}} + \boldsymbol{N}_{\mathrm{acc}} + \boldsymbol{N}_{\mathrm{mag}}$$

Accretion (spin-up) → $\boldsymbol{N}_{\mathrm{acc}}$

Disk-magnetosphere interaction (spin-down) → $\boldsymbol{N}_{\mathrm{mag}}$

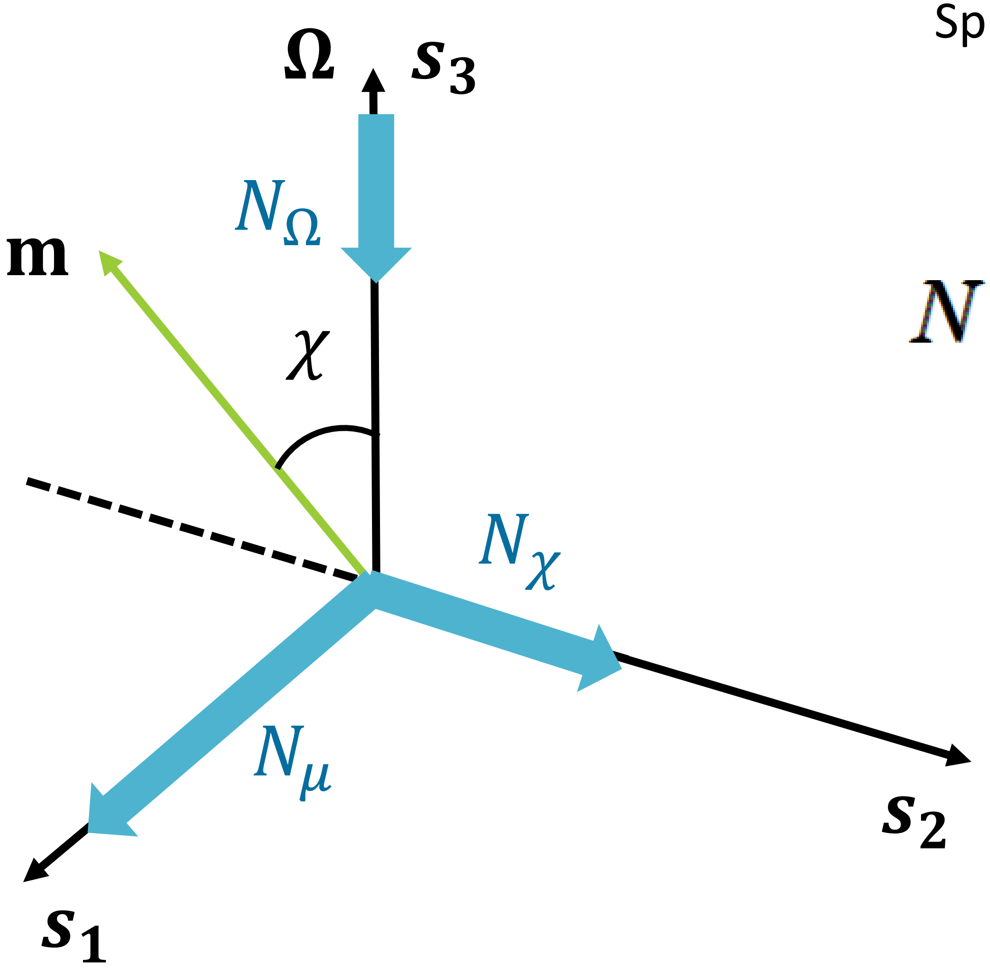


Spitkovsky, astro-ph/0603147



Parfrey et al., 1507.08627

Torque components



Magnetic angle evolution.
$\dot{\chi} < 0$ if $N_\chi < 0$ and vice-versa

Spin-down/spin-up

$$\boldsymbol{N} = N_\Omega \boldsymbol{s}_3 + N_\chi \boldsymbol{s}_2 + N_\mu \boldsymbol{s}_1$$

"Radiative precession."
Spin axis tends to precess around the magnetic axis with the frequency:

$$\boldsymbol{\omega}_{\mathrm{rad}} = \varepsilon_\mu \Omega \cos\chi \cdot \mathbf{m}$$

I.e. an "effective deformation": $\varepsilon_\mu = -\dfrac{N_\mu}{I\Omega^2 \sin\chi \cos\chi}$

Pulsar “anomalous” torque

$$N_{\rm psr} = -\mu^2/r_{\rm LC}^3$$

$$N_\mu = \frac{1}{3}\left(\frac{r_{\rm LC}}{r_*}\right) N_{\rm psr} \sin\chi \cos\chi$$

Beskin & Zheltoukhov, 1411.2107

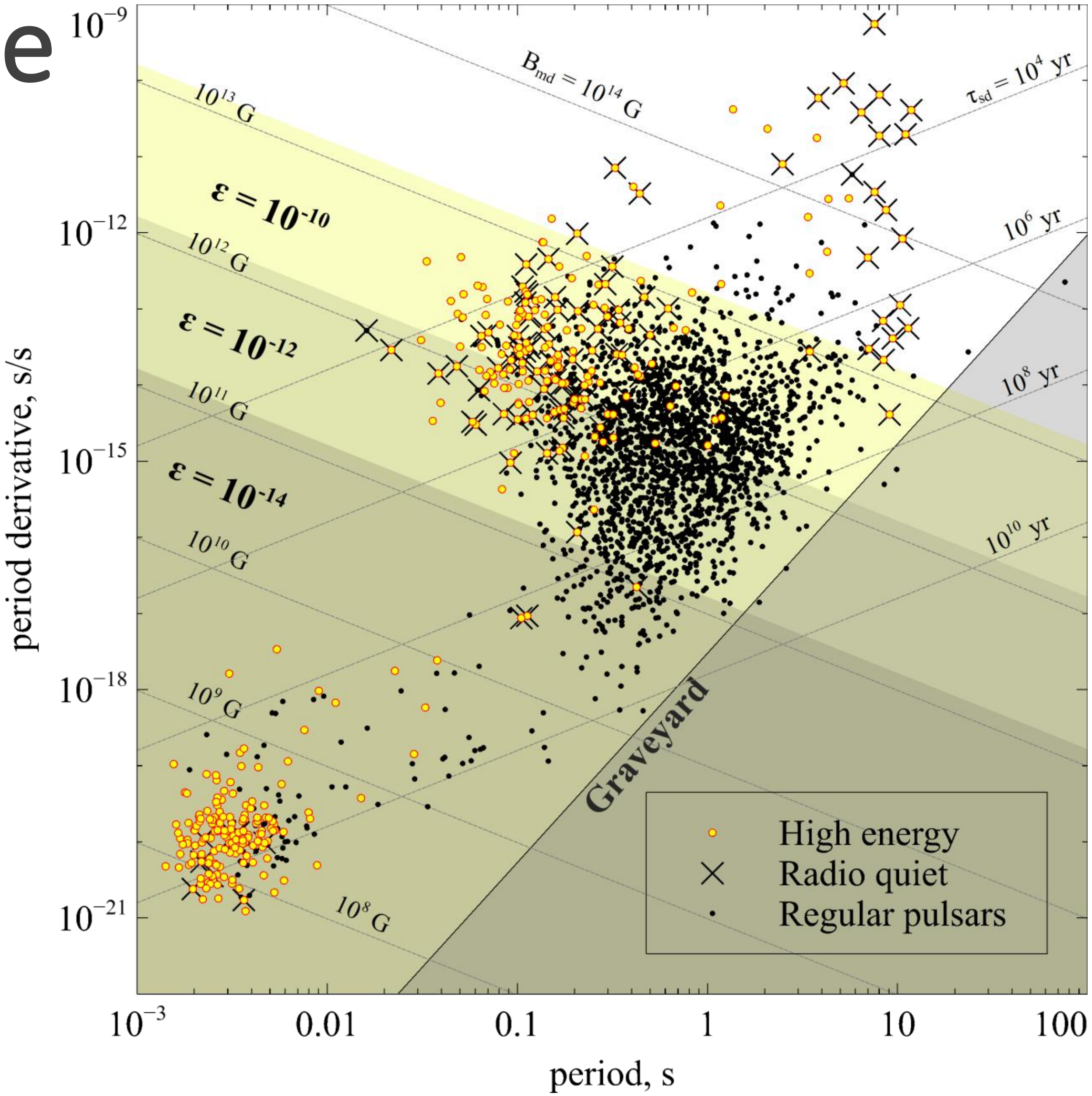
$$\varepsilon_\mu = -\frac{1}{3}\frac{\mu^2}{I R_{NS}\, c^2} \approx -1.1\cdot 10^{-12}\, B_{12}^2 I_{45}^{-1} R_{NS,12.5\rm km}^5$$

$$T_{\rm rad} = \frac{2\pi}{\varepsilon_\mu \Omega} \approx (2.8\cdot 10^4\ {\rm yr})\, P_s\, B_{12}^{-2}\, I_{45}\, R_{NS,12.5\rm km}^{-5}$$

$|\varepsilon_\mu| = \varepsilon$ corresponds to

$$B_{\rm md} = 2.25\cdot 10^{12}\sqrt{\varepsilon_{-12}}\ {\rm G},$$

where $B_{\rm md} = 3.2\cdot 10^{19}\sqrt{P\,\dot{P}}$ G



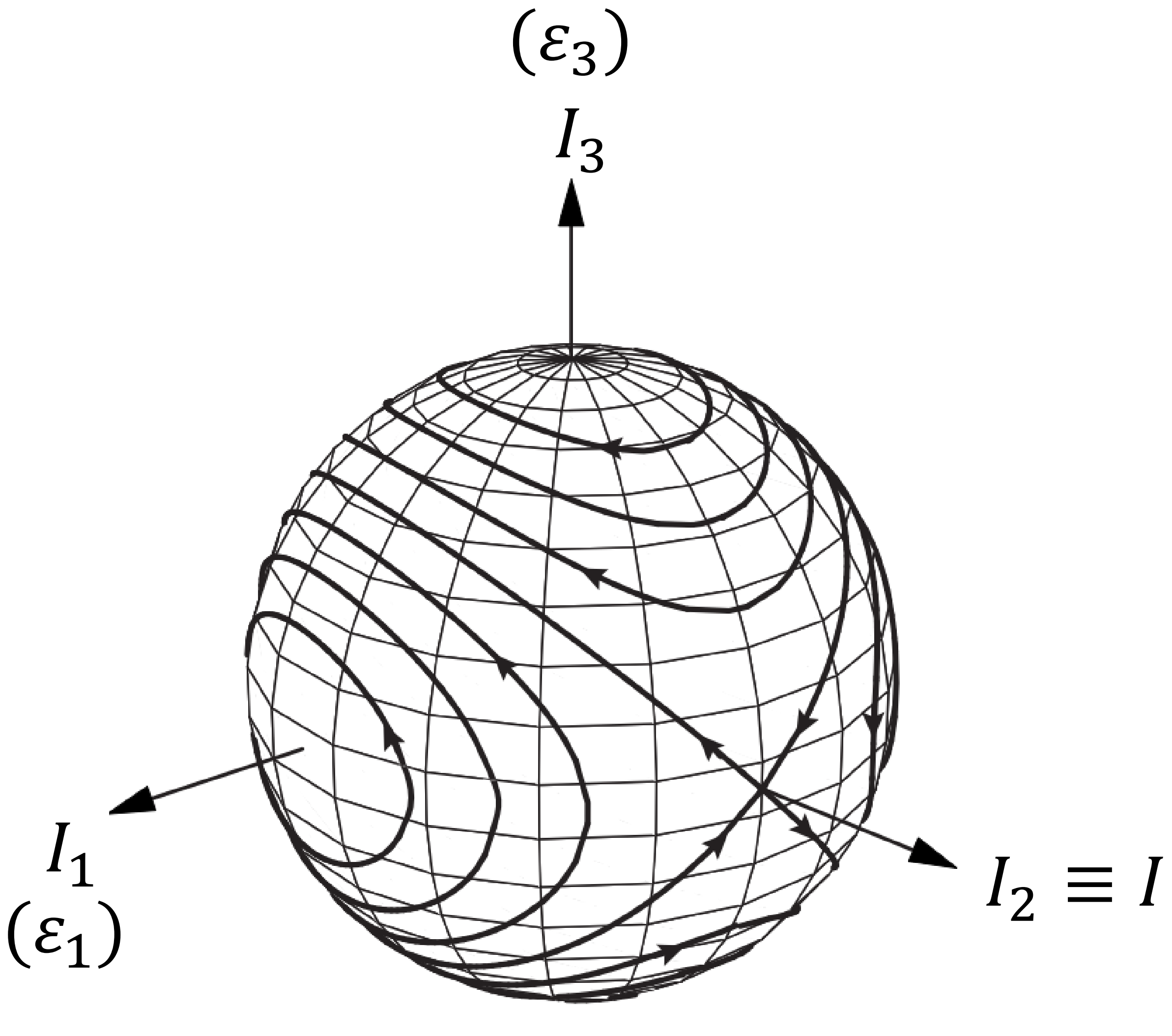
Precession

If deformation is small: $\varepsilon = \sqrt{\varepsilon_1^2 + \varepsilon_3^2} \ll 1,$

then Euler's equations in the rotating frame:

$$\dot{\mathbf{\Omega}} = (\boldsymbol{\omega} \times \mathbf{\Omega}) + \frac{\boldsymbol{N}}{I}$$

$$\boldsymbol{\omega} = (\varepsilon_1\Omega_1, 0, \varepsilon_3\Omega_3)^{\mathrm{T}}$$



$I_1 < I_2 < I_3$

Precession

If deformation is small: $\varepsilon = \sqrt{\varepsilon_1^2 + \varepsilon_3^2} \ll 1,$

then Euler's equations in the rotating frame:

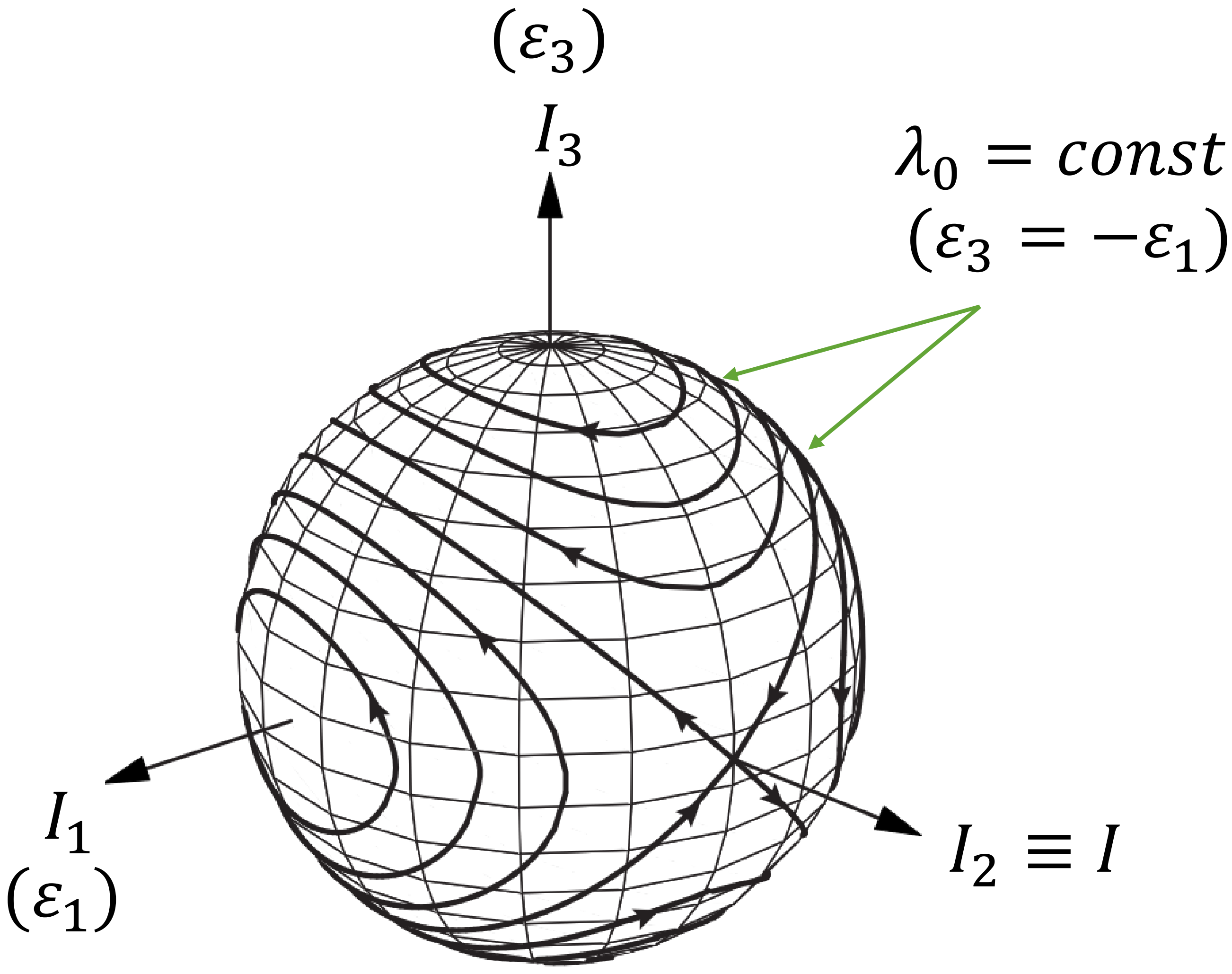
$$\dot{\boldsymbol{\Omega}} = (\boldsymbol{\omega} \times \boldsymbol{\Omega}) + \frac{\boldsymbol{N}}{I}$$

$$\boldsymbol{\omega} = (\varepsilon_1 \Omega_1, 0, \varepsilon_3 \Omega_3)^{\mathrm{T}}$$

The spin axis follows the lines of

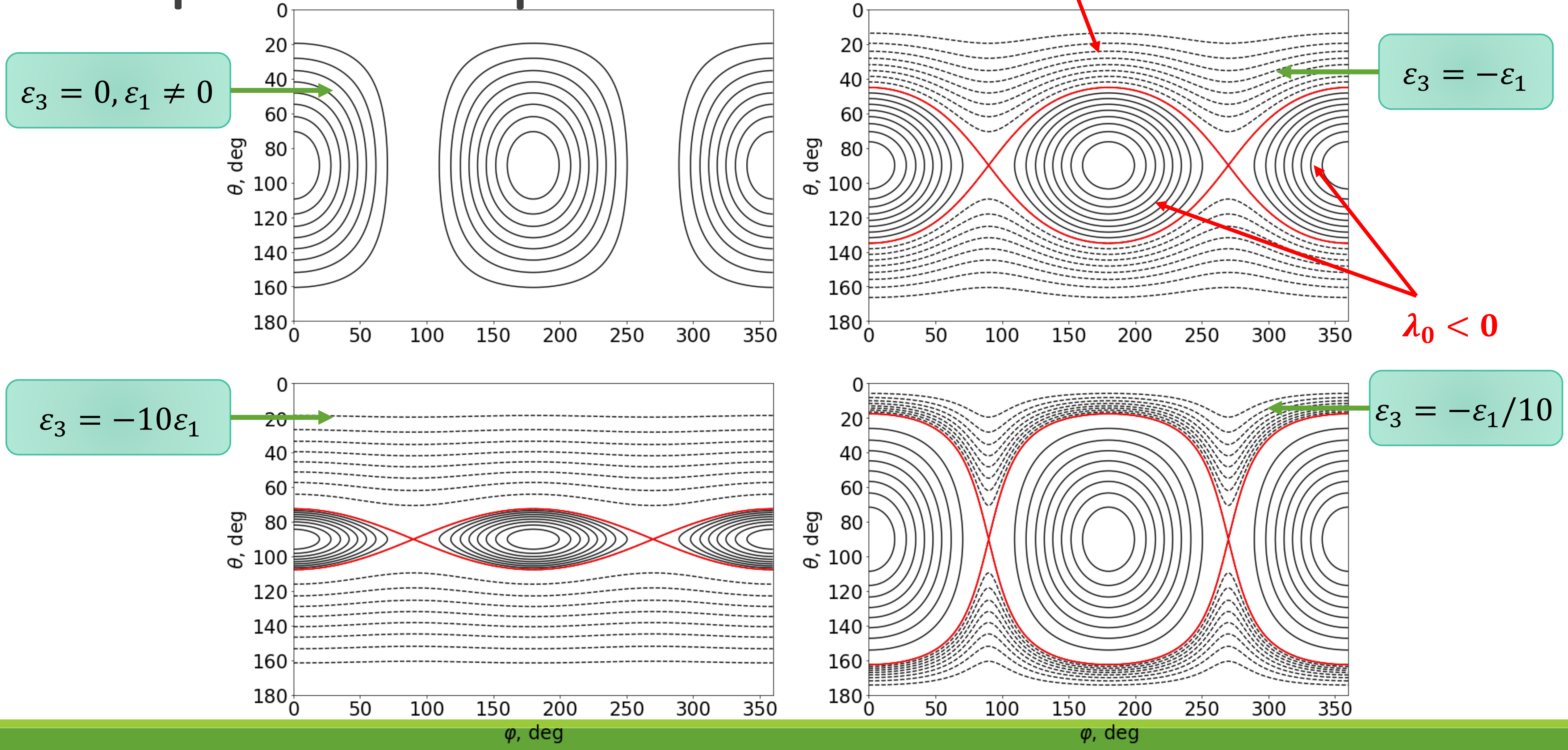
$$\lambda_0 = \frac{\boldsymbol{\Omega} \cdot \boldsymbol{\omega}}{\Omega^2} = const$$

with precessional period $2\pi/\omega$



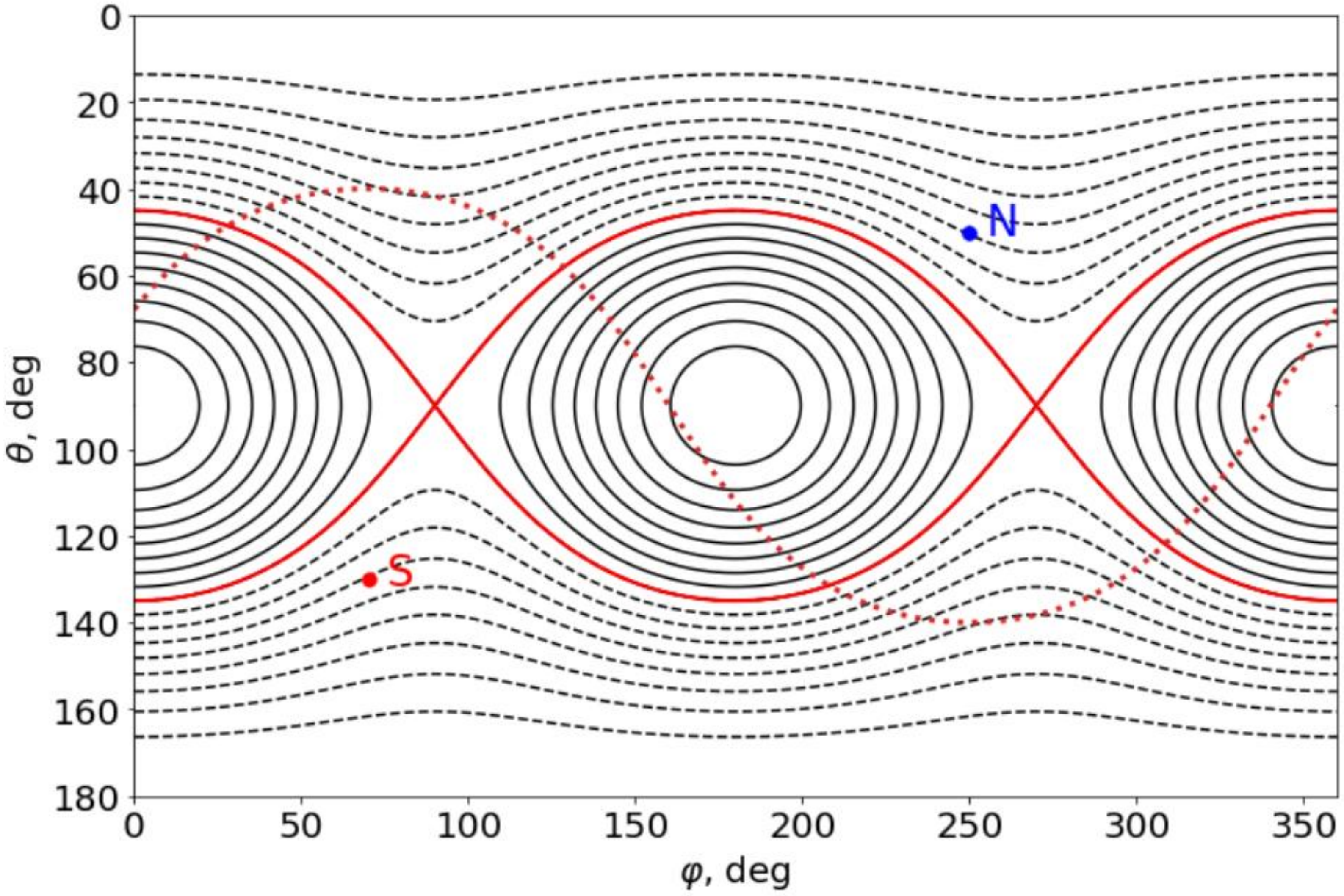
$I_1 < I_2 < I_3$

Free precession paths



Mixed precession: free + radiative

$\boldsymbol{N} = N_\Omega \boldsymbol{s}_3 + N_\chi \boldsymbol{s}_2 + N_\mu \boldsymbol{s}_1$



(Assuming $\varepsilon_1 = -\varepsilon_3$)

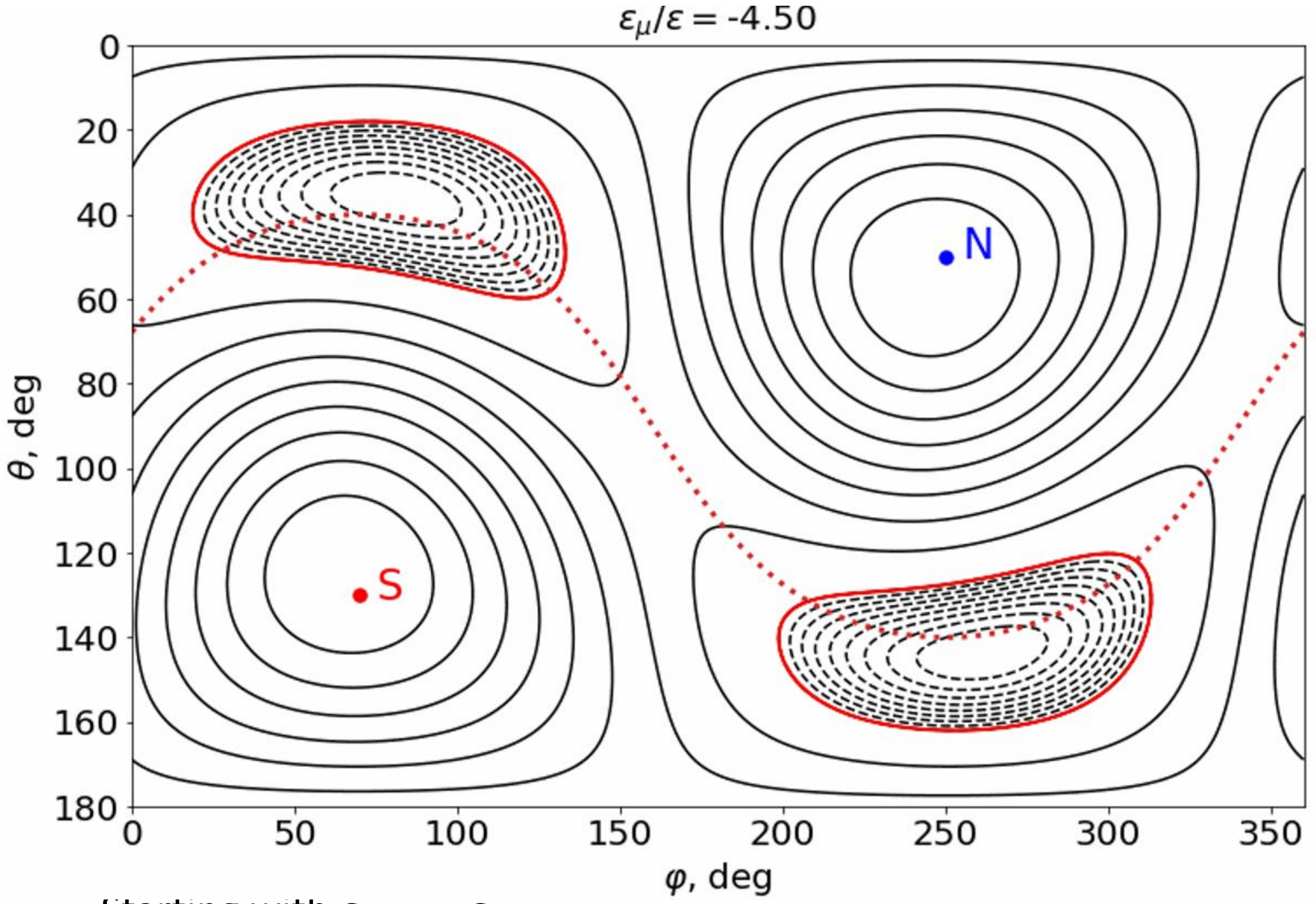
Mixed precession: free + radiative

$$\boldsymbol{N} = N_\Omega \boldsymbol{s}_3 + N_\chi \boldsymbol{s}_2 + N_\mu \boldsymbol{s}_1$$

$$\lambda = \frac{\boldsymbol{\Omega} \cdot \boldsymbol{\Omega}_\mathrm{p}}{\Omega^2} = const$$

$$\boldsymbol{\Omega}_\mathrm{p} = \boldsymbol{\omega} + \boldsymbol{\omega}_\mathrm{rad}$$

If $N_\chi = 0$, then spin axis precesses around the axis, which is **tilted** relative both inertia and magnetic axis.



Starting with $\varepsilon_1 = -\varepsilon_3$

Long-term evolution of an isolated NS

$P_0 = 20\ \mathrm{ms}$

$\mu_0 = 5 \cdot 10^{30}\ \mathrm{G \cdot cm^3}$

$\chi_0 = 40°$

$\varepsilon_3 = -\varepsilon_1 = 2 \cdot 10^{-12}$

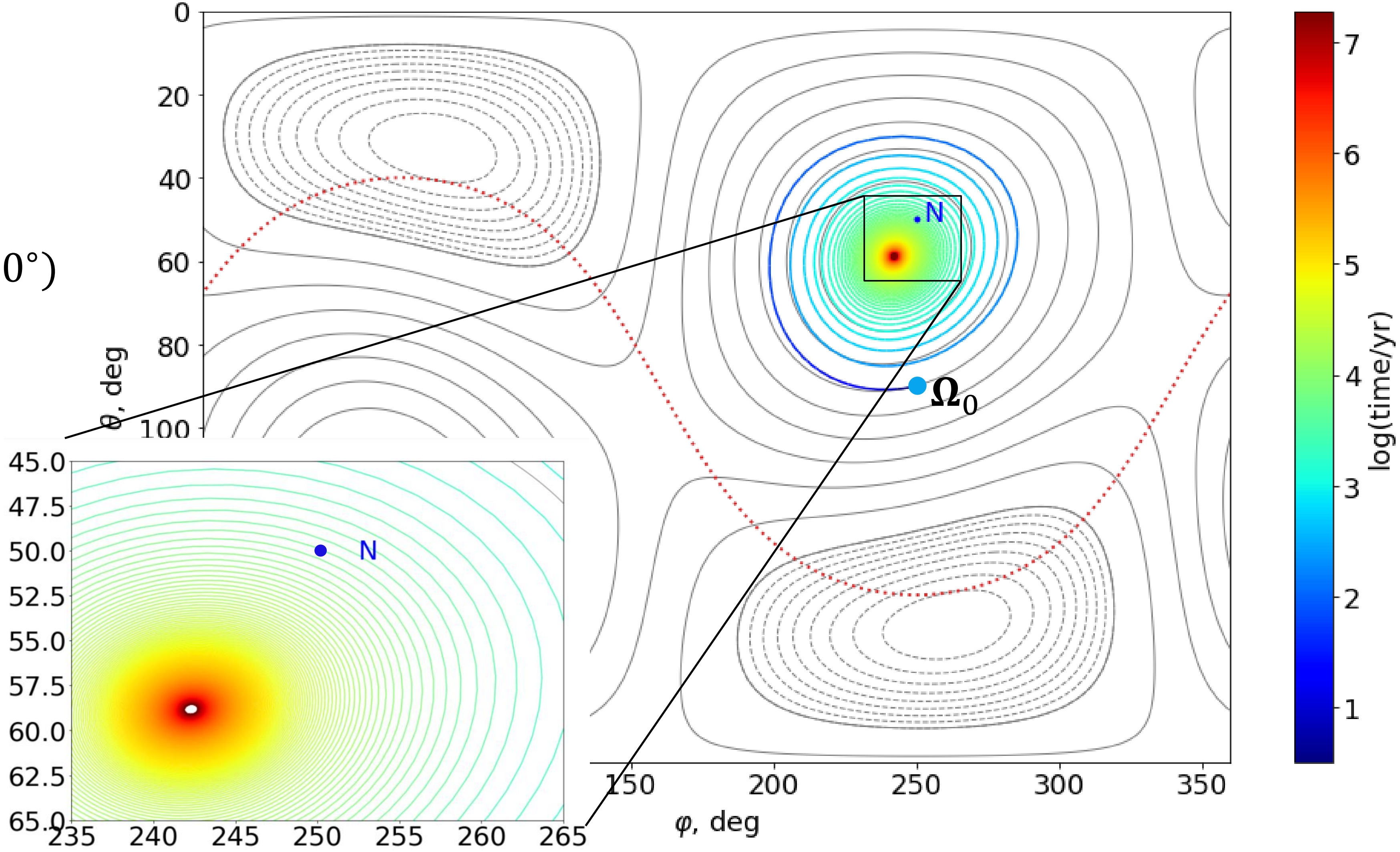
Magnetic axis $(\varphi_\mathrm{m}, \theta_\mathrm{m}) = (250°, 50°)$

Constant magnetic field

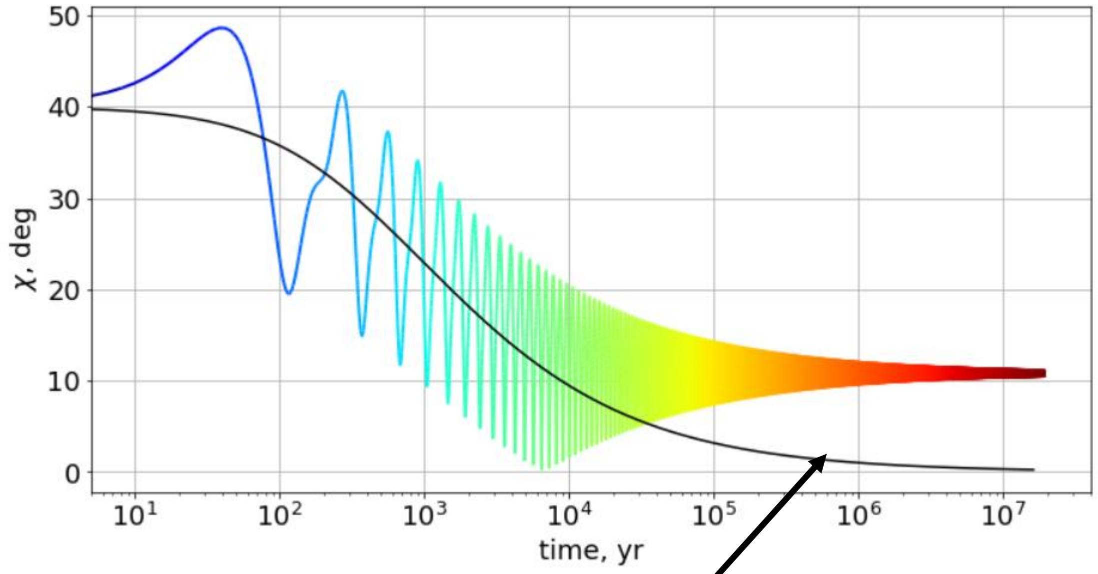
$\boldsymbol{N} = \boldsymbol{N}_\mathrm{psr}$

$$\frac{d\lambda}{dt} = \frac{N_\chi}{I\Omega^2} \sin \chi_1$$

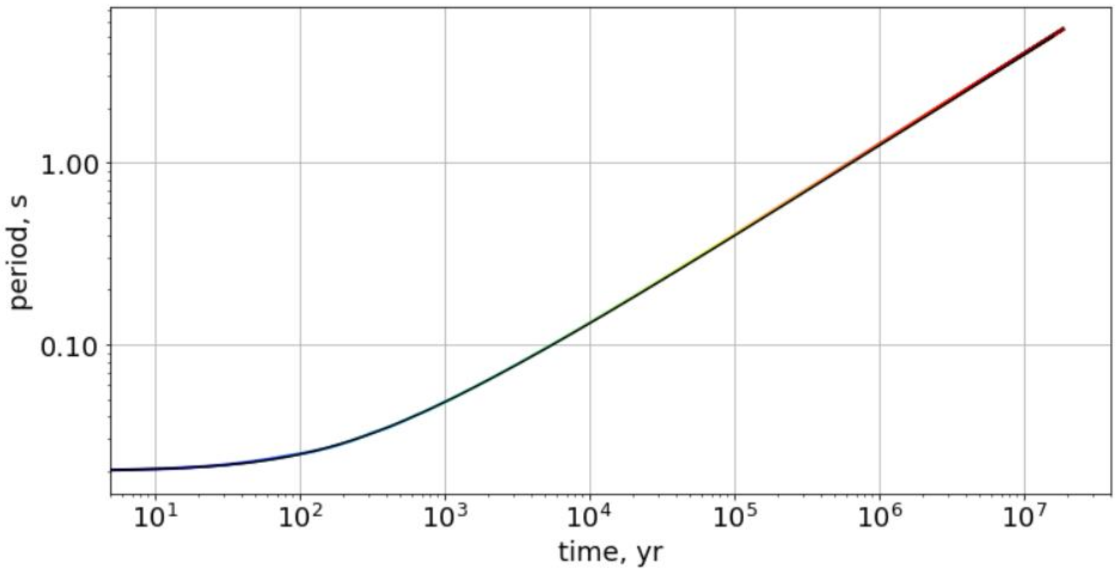
χ_1 is the angle between $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}_\mathrm{p}$



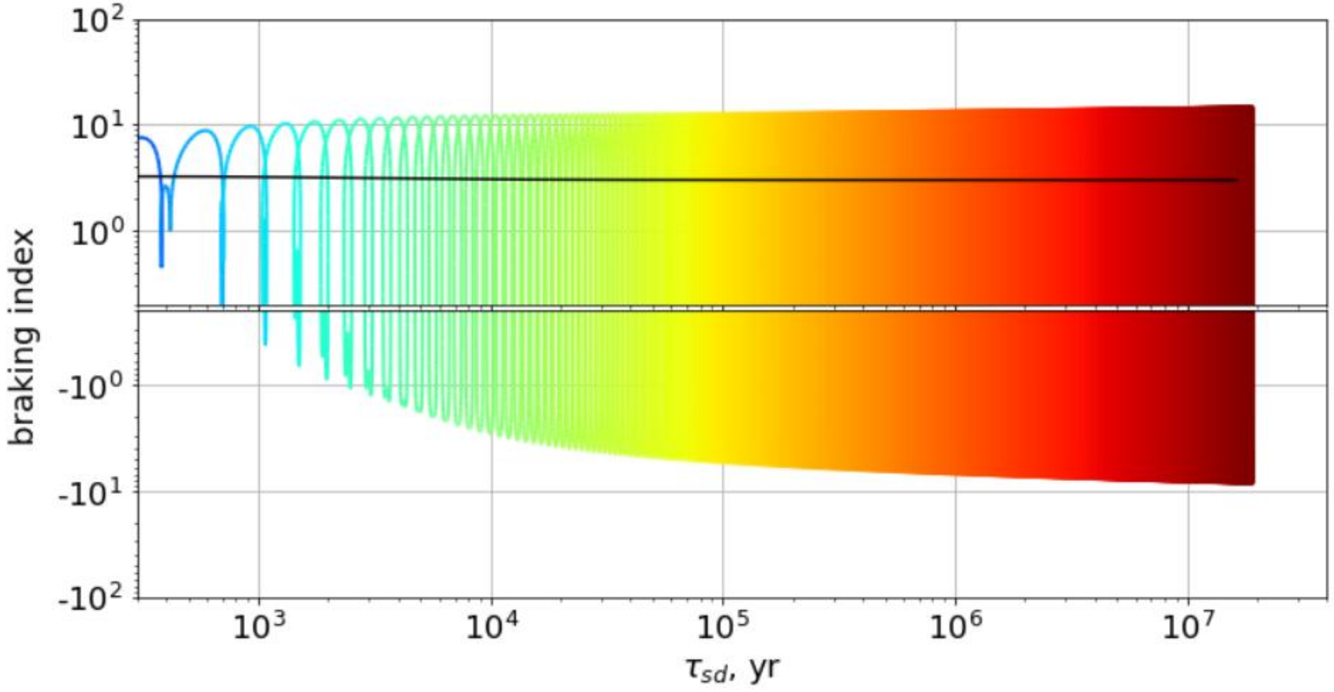
AB, Abolmasov & Levinson, in prep.



Spherical star



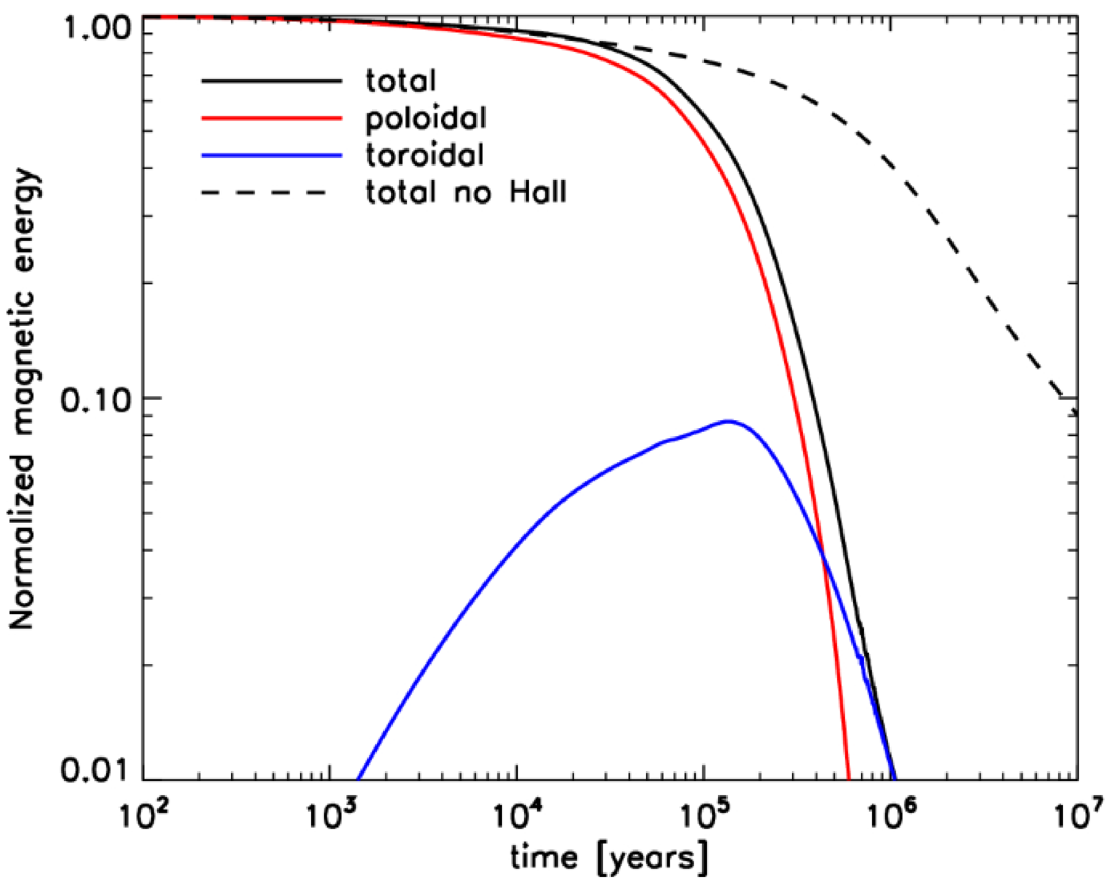
See also Arzamasskiy et al., 1504.06626



$$\dot{\Omega} \propto -B^2\Omega^3(k_0 + k_1 \sin^2\chi)$$

$$n_{br} = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 3..3.25 \text{ for a spherical pulsar.}$$

INS magnetic field decay



Pons & Vigano, 1911.03095

$$B(t) = B_0 \frac{\exp\left(-\frac{t}{\tau_{\rm Ohm}}\right)}{1 + \frac{\tau_{Ohm}}{\tau_{Hall}}\left[1 - \exp\left(-\frac{t}{\tau_{\rm Ohm}}\right)\right]}$$

Aguilera et al., 0710.0854

$\tau_{\rm Ohm} = 5 \cdot 10^5$ yr

$\tau_{\rm Hall} = 10^4\, B_{0,15}^{-1}$ yr

"Hall attractor" at $B = 0.05 B_0$

also e.g.

Igoshev & Popov, 1507.07962

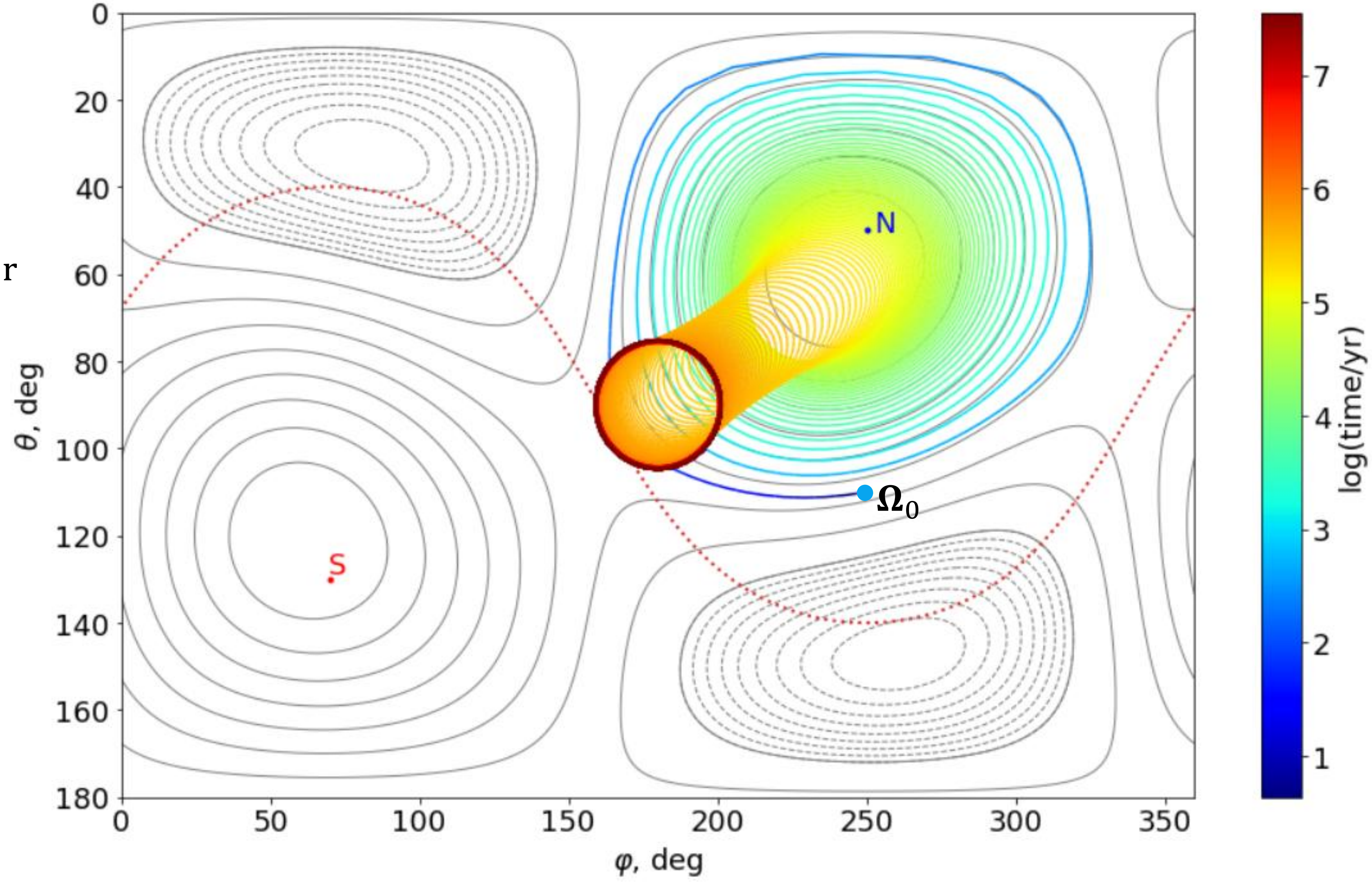
Afonina, AB & Popov, 2310.14844

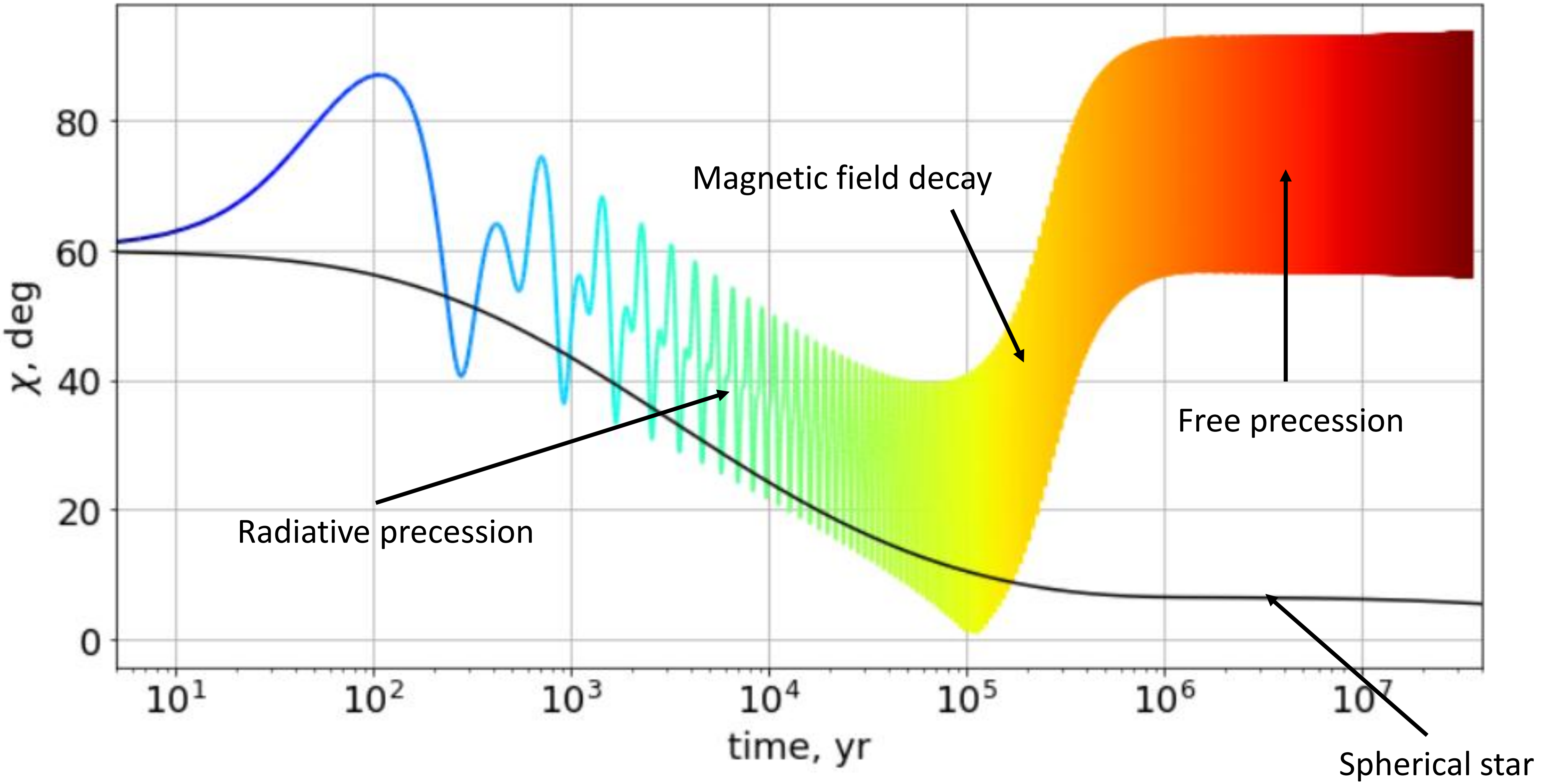
Igoshev et al., 2109.05584

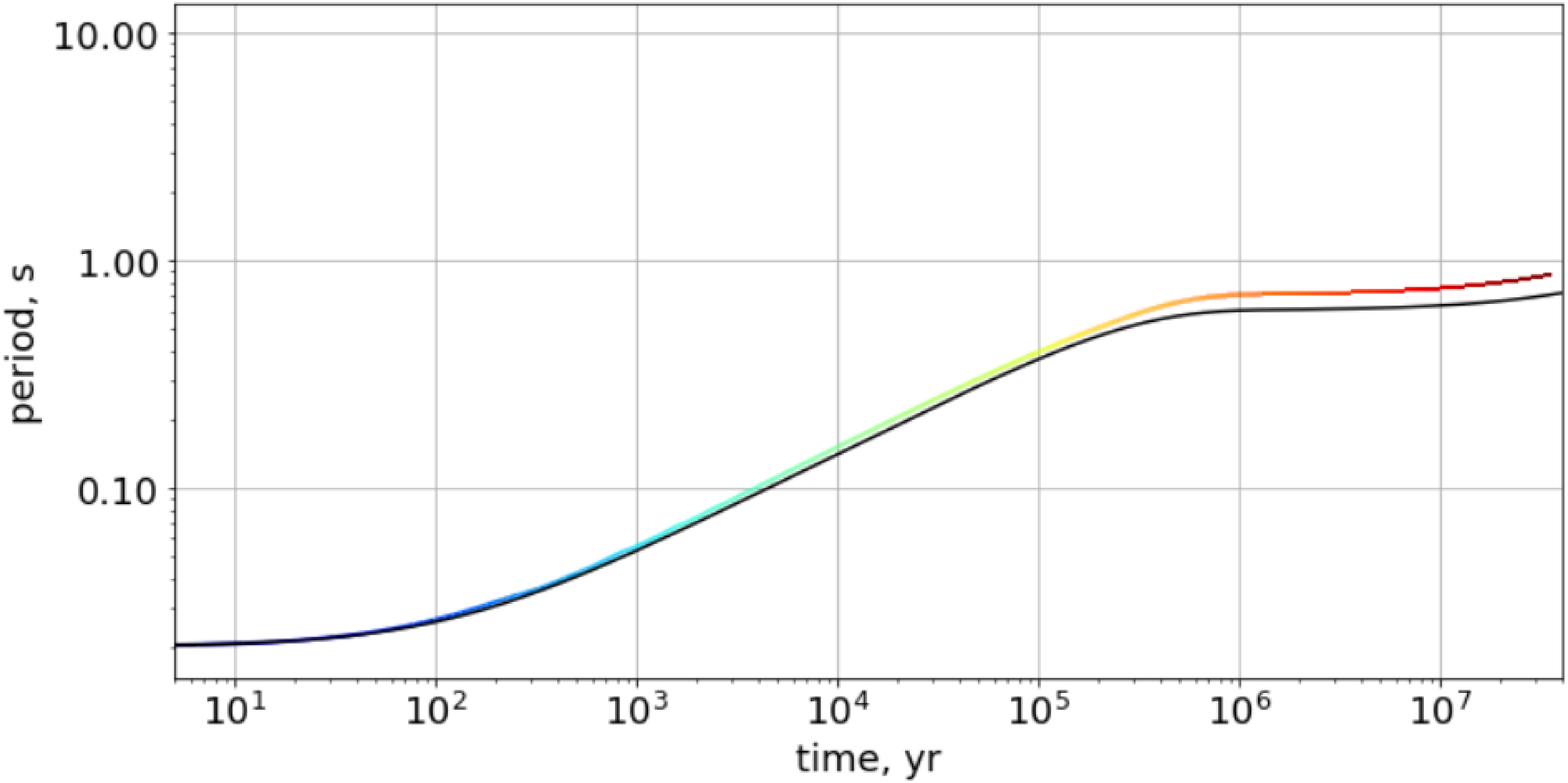
$\chi_0 = 60^\circ$

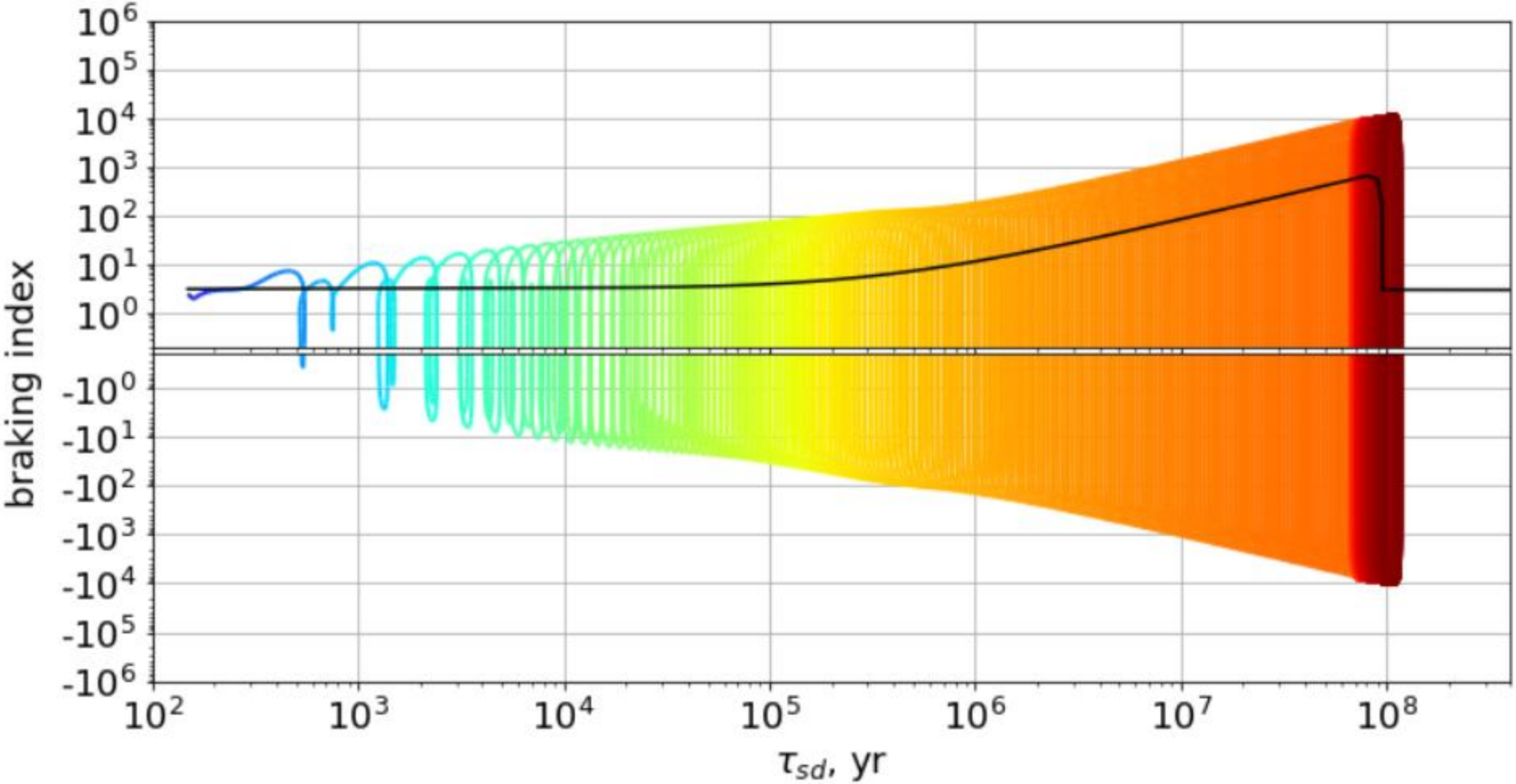
$B = B(t)$

$\boldsymbol{N} = \boldsymbol{N}_{\rm psr}$



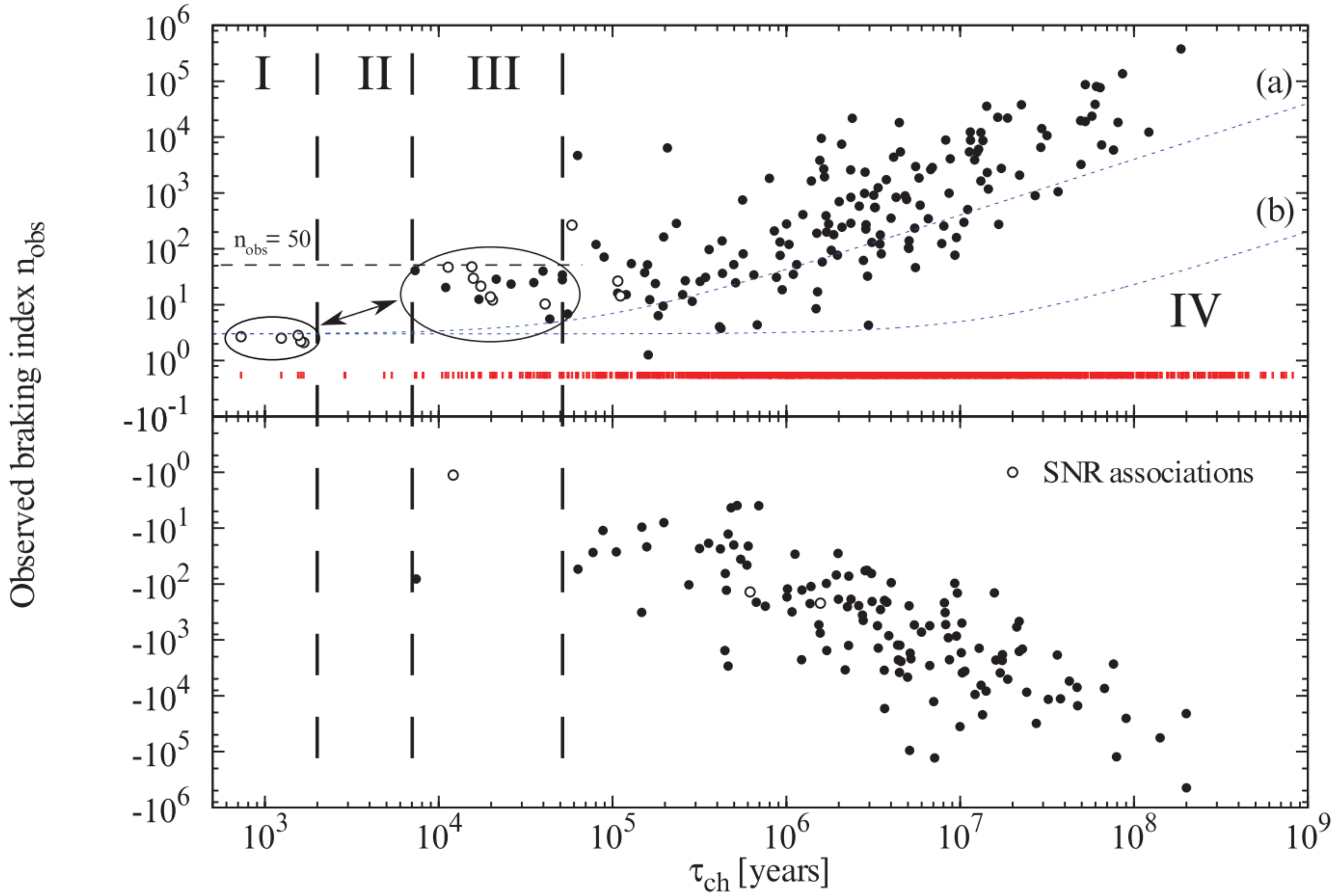

80
60
40
20
0
χ, deg
10^1
10^2
10^3
10^4
10^5
10^6
10^7
time, yr
Magnetic field decay
Free precession
Radiative precession
Spherical star


10.00
1.00
0.10
period, s
10^1
10^2
10^3
10^4
10^5
10^6
10^7
time, yr



braking index
10^6
10^5
10^4
10^3
10^2
10^1
10^0
-10^0
-10^1
-10^2
-10^3
-10^4
-10^5
-10^6
10^2
10^3
10^4
10^5
10^6
10^7
10^8
τ_{sd}, yr

Braking indices of isolated pulsars



Could braking indices be the probes of NSs deformation and magnetic field evolution?

AB, Beskin & Karpov, 1105.5019

Accreting neutron star

$$N_\Omega = -\left(\frac{r_{\rm LC}}{r_{\rm m}}\right)^2 \frac{\mu^2}{r_{\rm LC}^3}(k_0 + k_1 \sin^2\chi) + N_{\rm mag} + N_{\rm acc}\cos\alpha$$

$$N_{\rm mag} \sim -\frac{\mu^2}{r_{\rm co}^3}$$

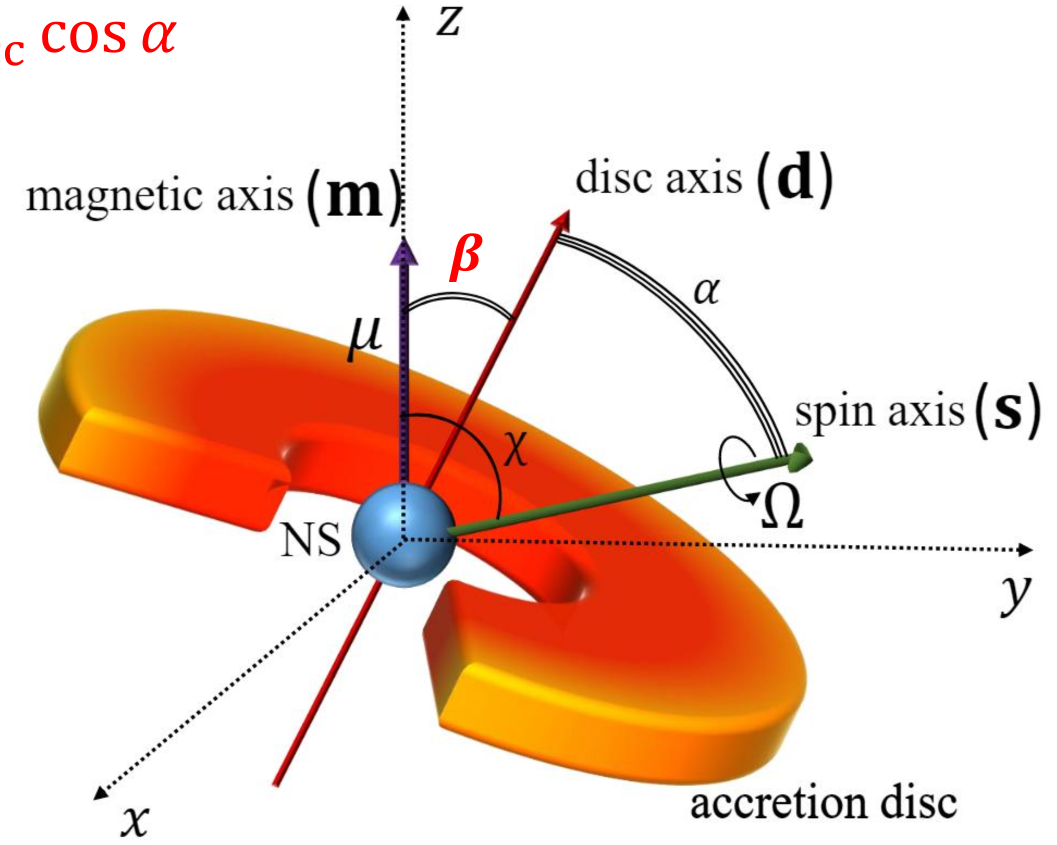
$$\boldsymbol{N}_{\rm acc} = \dot{M} f(\beta)\sqrt{GM_* r_{\rm m}}\,\boldsymbol{d}$$

$$f(\beta) = A(\eta, \alpha, \chi)\cdot(1 - \eta\cos^2\beta)$$

Spin-up torque is modulated with the magnetic axis tilt with respect to the disc axis.

e.g. Kulkarni & Romanova, 1303.4681

Romanova et al., 2012.10826



AB & Abolmasov, 2105.00754

Accreting neutron star

- Magnetic angle evolution of a spherical accreting star:

$$N_\chi = -\left(\frac{r_{\rm LC}}{r_{\rm m}}\right)^2 k_2 \frac{\mu^2}{r_{\rm LC}^3} \sin\chi\cos\chi + \eta A \cdot N_{\rm acc} \sin^2\alpha\cos\alpha\sin\chi\cos\chi$$

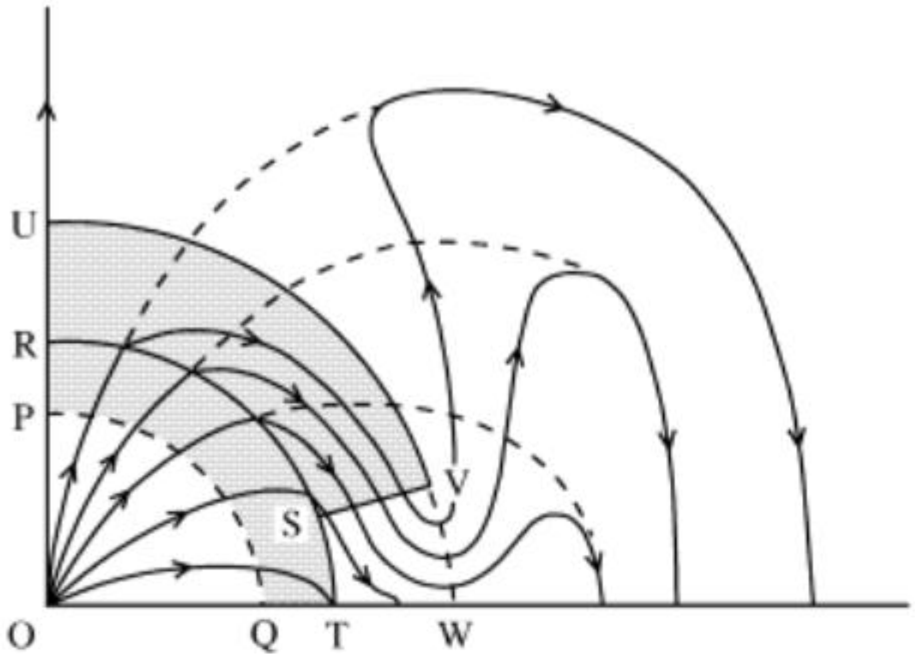
- Fast radiative precession:

$$\varepsilon_\mu = -4 \cdot 10^{-10}\, B_{12}^{6/7}\, P_s^2\, \dot{M}_{-9}^{4/7}\, I_{45}^{-1}\, R_{NS,12.5\rm km}^5$$

$$T_{\rm rad} \approx (80\ {\rm yr})\, P_s^{-1}\, B_{12}^{-6/7}\, \dot{M}_{-9}^{-4/7}\, I_{45}\, R_{NS,12.5\rm km}^{-5}$$

$$r_{\rm m} = \frac{r_{\rm A}}{2}, \qquad \dot{M}_{-9} = \frac{\dot{M}}{10^{-9}\, M_\odot\, {\rm yr}^{-1}}$$

Dipole field burial



(Melatos & Phinney, 2001, PASA, 18, 421)

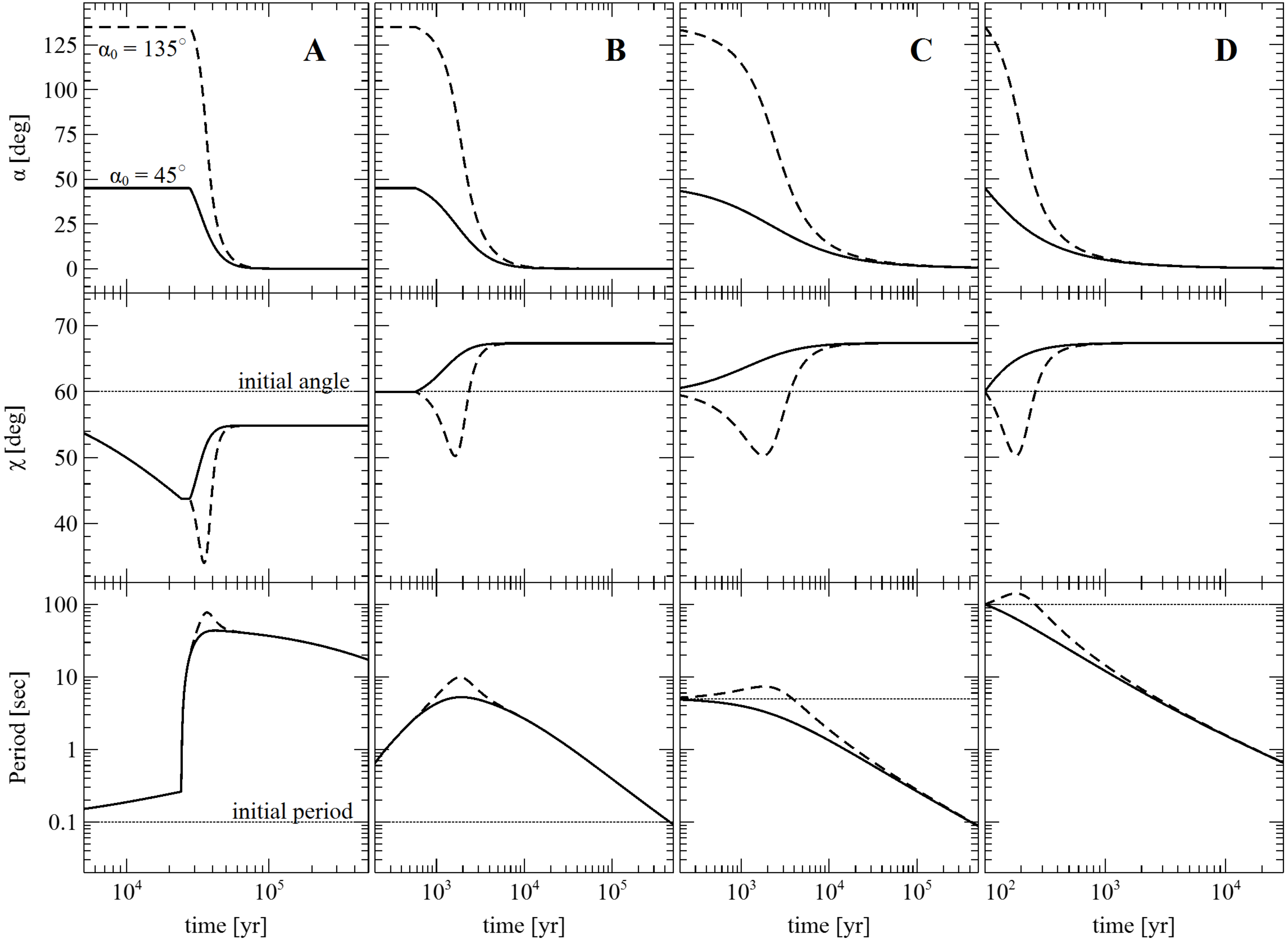
$$\mu = \mu_0 \left(1 + \frac{\Delta M_*}{\mathcal{M}}\right)^{-14/11}$$

where

$$\mathcal{M} = 1.1 \times 10^{-5} \dot{M}_1^{1/7} \mu_{0,30}^{3/14} R_{12.5}^3 \ M_\odot$$

So that:

$$\tau_{\rm bur} = -\frac{\mu}{\dot{\mu}} \approx 3.4 \times 10^3 \dot{M}_1^{-6/7} \mu_{0,30}^{3/14} R_{12.5}^3 \ {\rm yr}$$

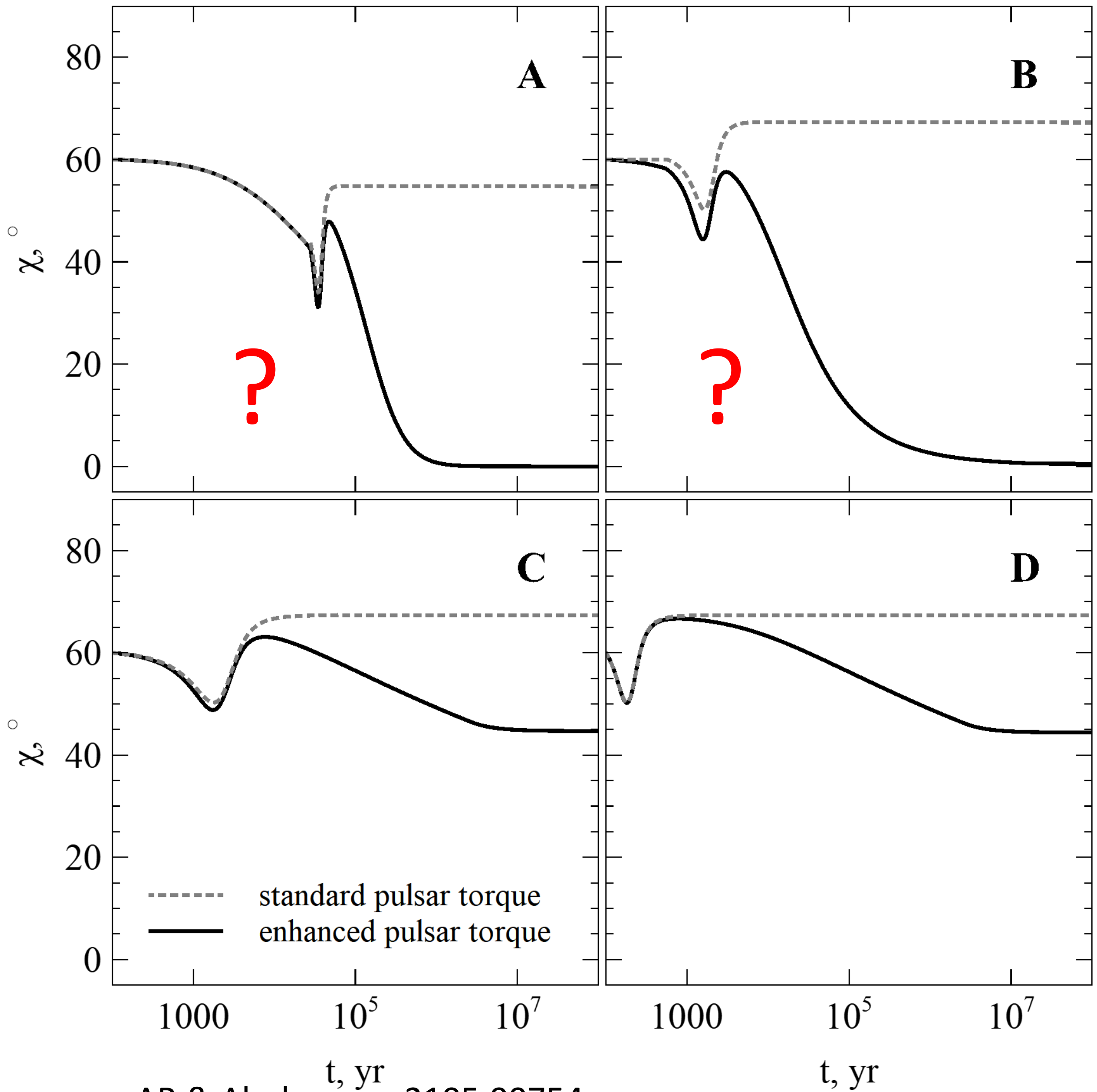


Model	P_0, sec	$\mu_{0,30}$	$\dot{M}_1$
A	0.1	5.0	0.01
B	0.1	5.0	1.0
C	5.0	1.0	1.0
D	100.0	1.0	1.0

$\chi_0 = 60°, \alpha_0 = 45°$ or $135°$

Evolution of a spherical accreting star **without pulsar torque enhancement.**

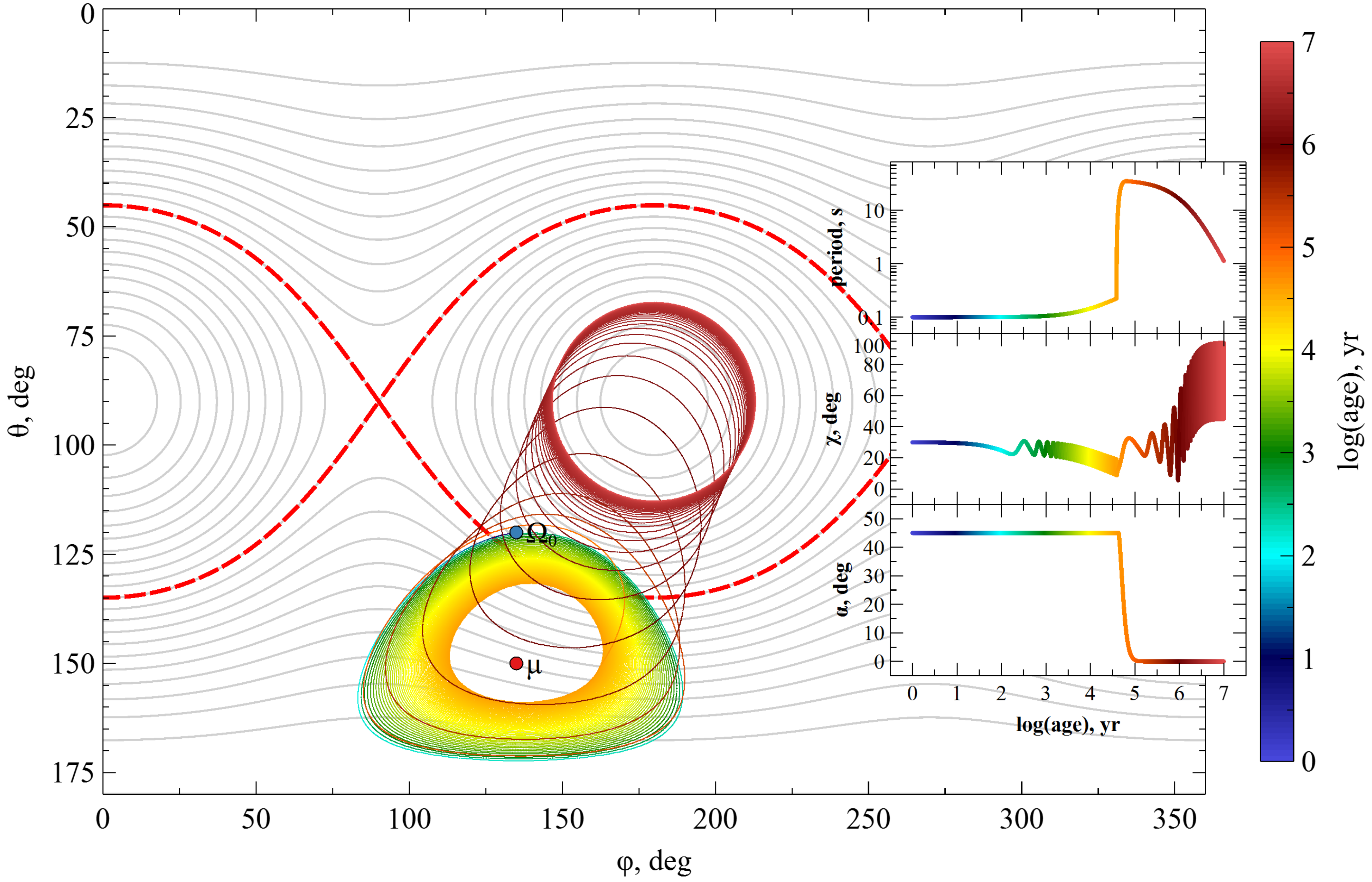
AB & Abolmasov, 2105.00754

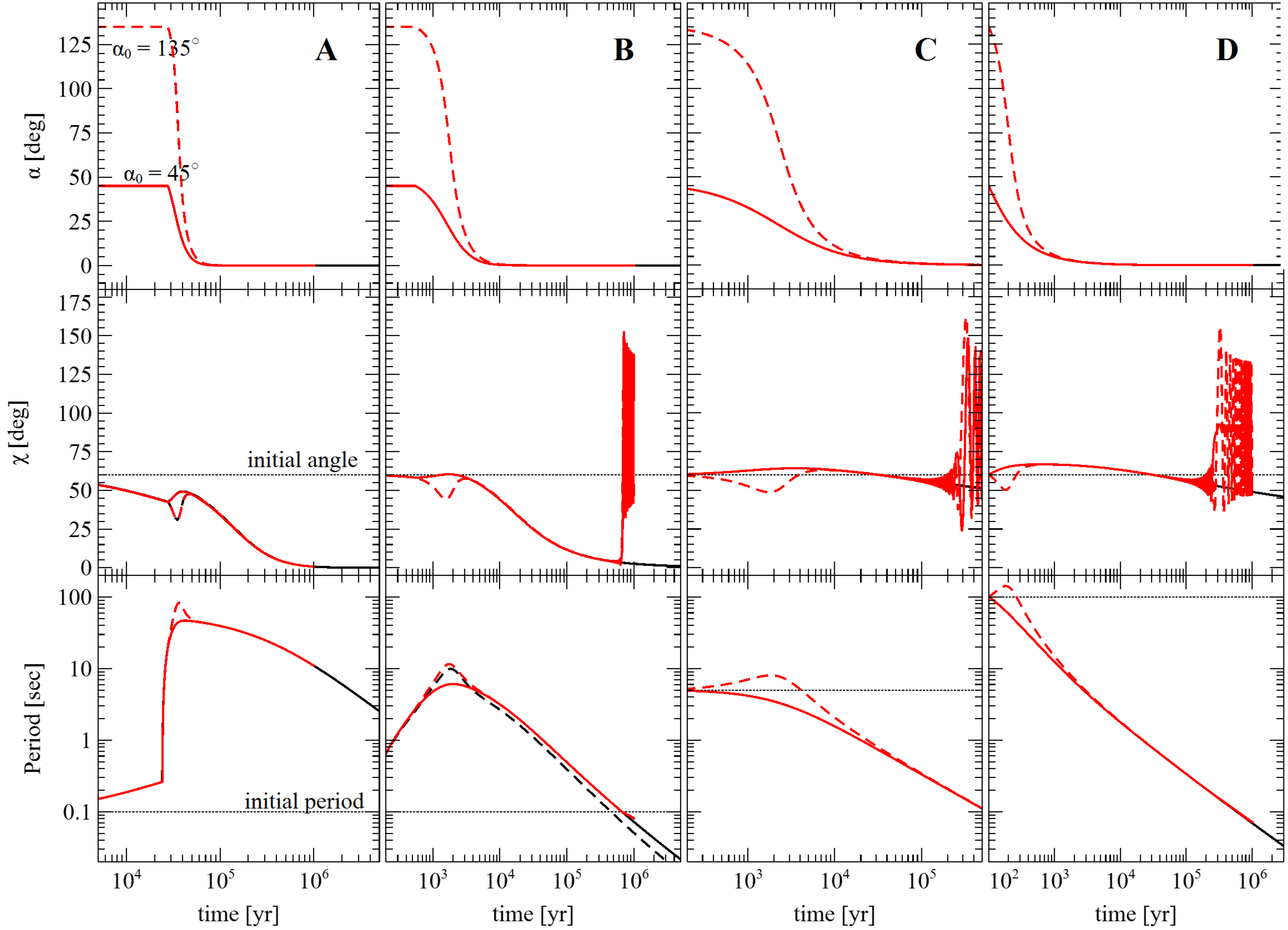


AB & Abolmasov, 2105.00754

Model	P_0, sec	$\mu_{0,30}$	$\dot{M}_1$
A	0.1	5.0	0.01
B	0.1	5.0	1.0
C	5.0	1.0	1.0
D	100.0	1.0	1.0

… and with pulsar torque enhancement.


θ, deg
φ, deg
Ω_0
μ
period, s
χ, deg
α, deg
log(age), yr
log(age), yr



Model	P_0, sec	$\mu_{0,30}$	$\dot{M}_1$
A	0.1	5.0	0.01
B	0.1	5.0	1.0
C	5.0	1.0	1.0
D	100.0	1.0	1.0

$\chi_0 = 60°, \alpha_0 = 45°$ or $135°$

Evolution of a triaxial ($\varepsilon = 10^{-13}$) accreting star. **With pulsar torque enhancement.**

Conclusions and *issues*

- **Even a weak star deformation $\varepsilon \sim 10^{-14}..10^{-10}$ will affect long-term evolution of its magnetic angle;**

- **Spin of a deformed star tends to align with an axis tilted with respect to the magnetic one;**

- **Magnetic field decay could lead to magnetic orthogonalization of a deformed star.**

- **Pulsar braking indices can be understood assuming the evolution of a deformed star.**

However:

- *How does the 'anomalous torque' works for an accreting NSs?*

- *Does torque enhancement works for magnetic angle evolution in the same way as for spin-down?*

- *How does superfluid interior affect long-term rotation of a deformed star?*

Thank you! ☺