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Fitting all spectra and light curves of gamma-ray pulsars with an austere synchro-curvature model

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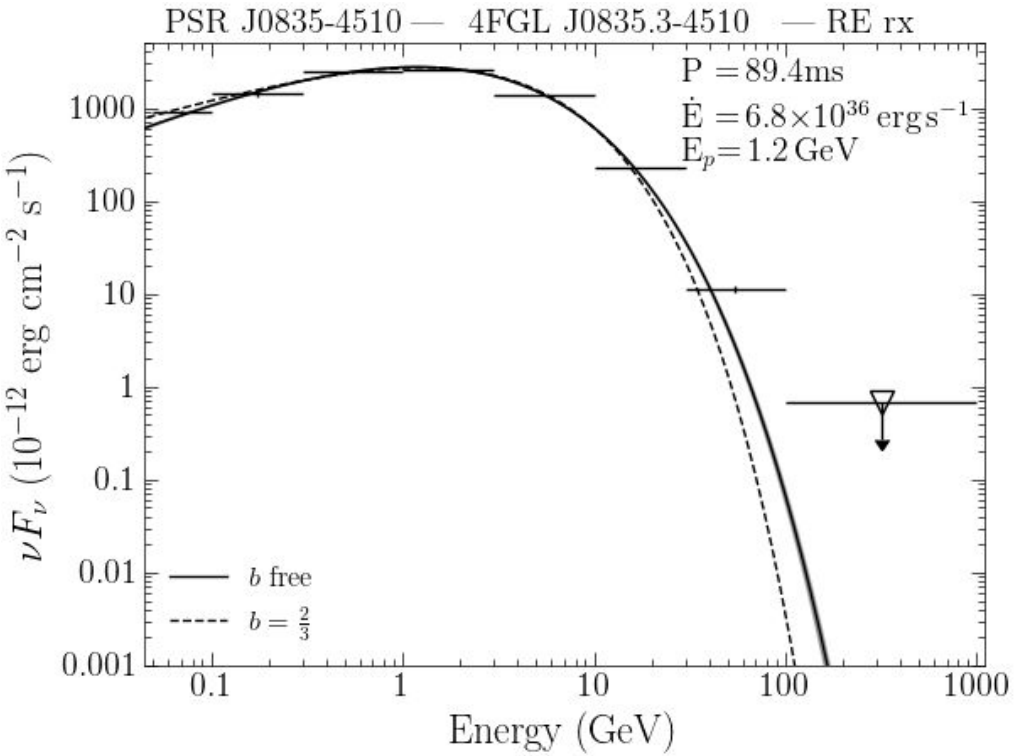
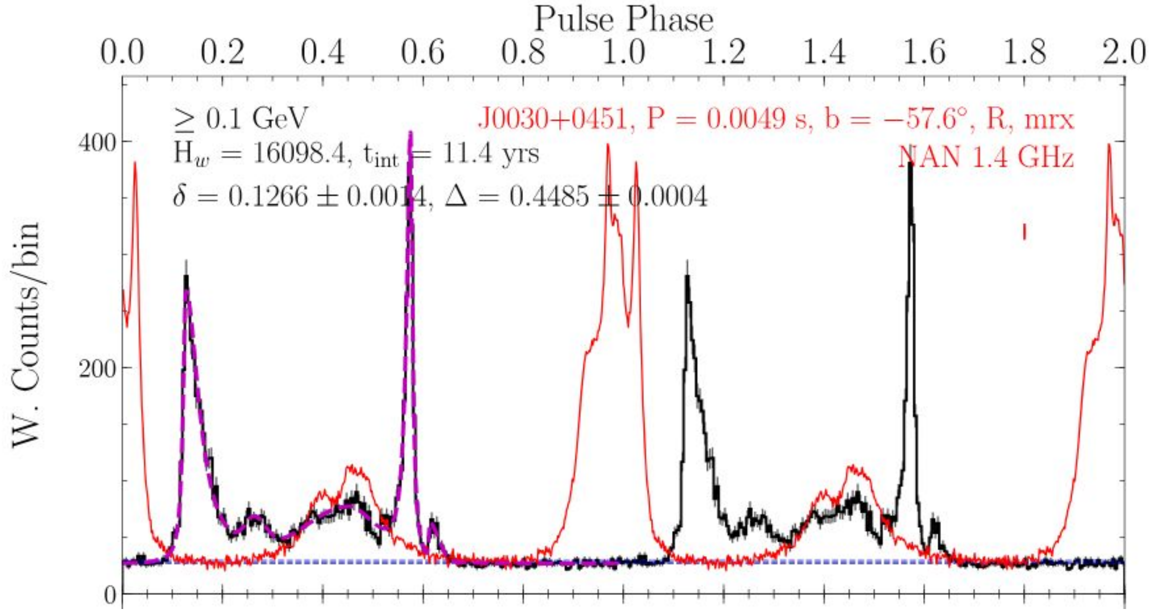
Feeling the pull and pulse of relativistic magnetospheres Workshop

6-11th April 2025 - Les Houches, France

Motivation of the project

- Models of high-energy emission of pulsars can be tested with e.g. the data of the gamma-ray emitting pulsars released in the 3PC (Smith et al. (2023))





- Our goal is to reproduce observational data with a model that contains simple but realistic physics and is computationally affordable

Spectral model: particle dynamics

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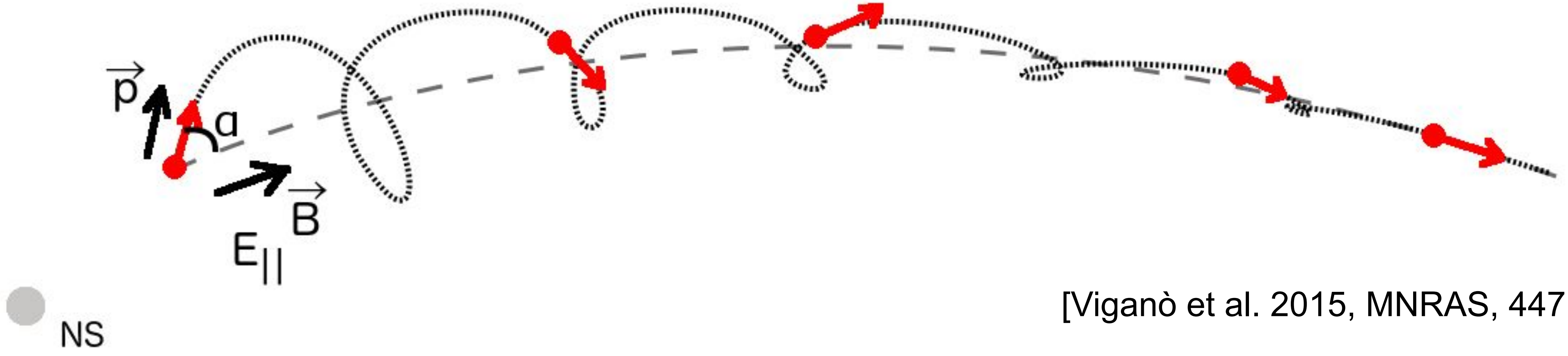
- We follow the dynamics of the emitting particles, ruled by electric acceleration and synchro-curvature losses and with two free parameters involved: $E_{||}$, b

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \mathrm{Ze}\, E_{\|}\hat{b} - \frac{P_{\mathrm{sc}}}{v}\,\hat{p} \qquad B(x) = B_\star\left(\frac{R_\star}{x}\right)^b \qquad r_c(x) = R_{\mathrm{lc}}\left(\frac{x}{R_{\mathrm{lc}}}\right)^\eta$$

- Solving the equation of motion gives the evolution of the relativistic momentum and of the Lorentz factor Γ and pitch angle α



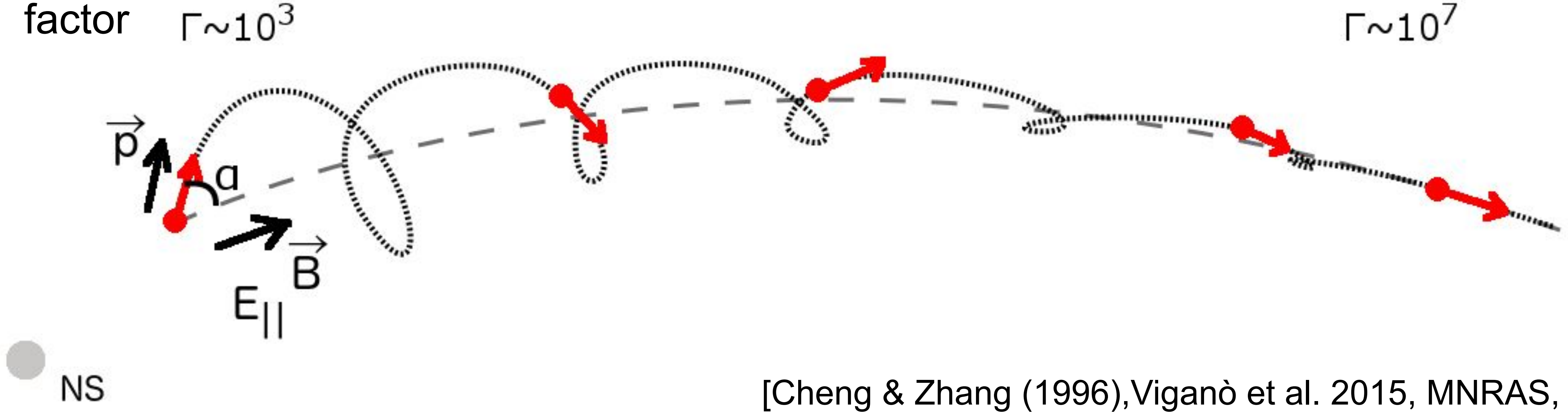
[Viganò et al. 2015, MNRAS, 447, 1164]

Spectral model: emission

- Synchro-curvature formulae gives the emission of the particles all along the trajectory, which convolved with an effective particle distribution gives the total emission from the region

$$\frac{dP_{\rm sc}}{dE} = \frac{\sqrt{3}e^2\Gamma y}{4\pi\hbar r_{\rm eff}}[(1+z)F(y) - (1-z)K_{2/3}(y)] \qquad \frac{{\rm d}N_{\rm e}}{{\rm d}x} = \left[\frac{N_0}{x_0(1-{\rm e}^{-(x_{\rm out}-x_{\rm in})/x_0})}\right]{\rm e}^{-(x-x_{\rm in})/x_0}$$

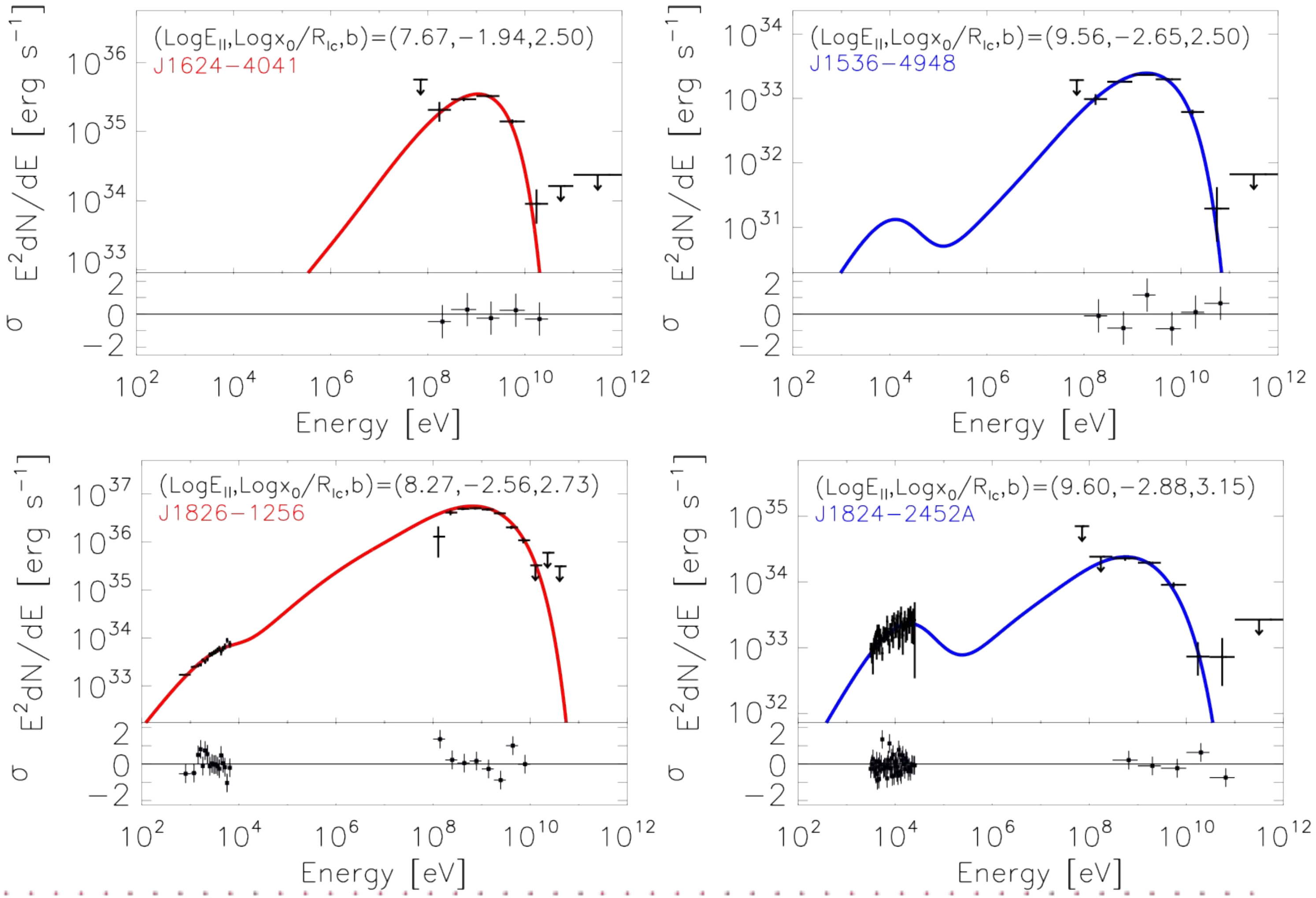
- We produce theoretical spectra with just three free parameter ($E_{||}$, b, x_0) and a normalization factor



[Cheng & Zhang (1996),Viganò et al. 2015, MNRAS, 447, 1164]

Spectral fitting

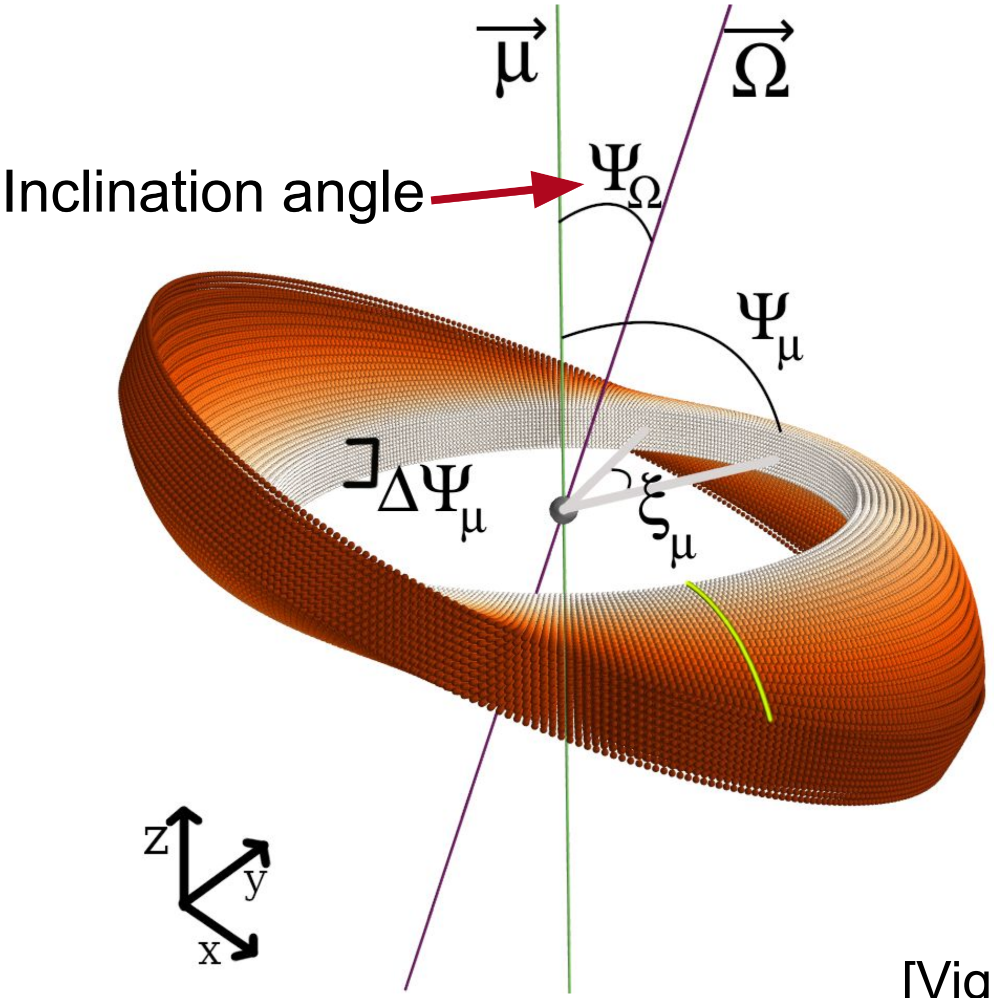
[Viganò et al. 2015, MNRAS, 453, 2599;
Torres 2018, Nat. Astron., 2, 247;
Torres et al. 2019, MNRAS, 489, 5494;
Íñiguez-Pascual, Viganò & Torres 2022, MNRAS, 516, 2475;
Íñiguez-Pascual, Torres & Viganò 2025, (submitted)]

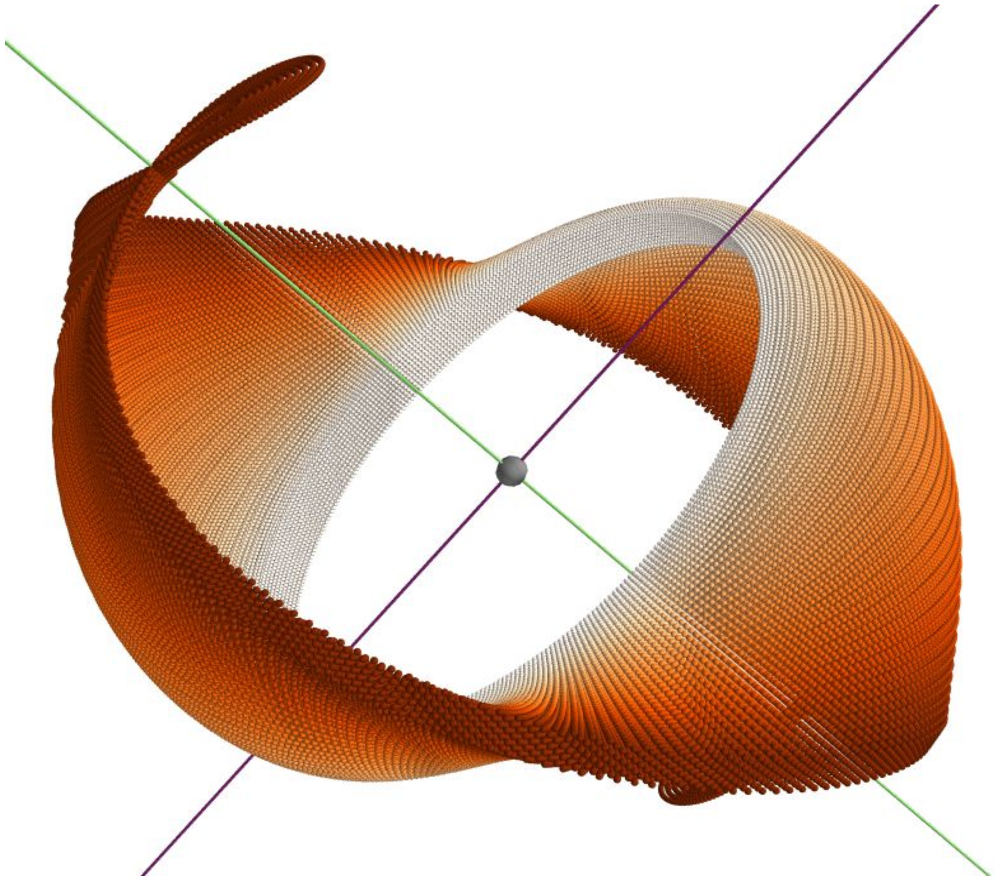


- The model successfully fitted the 117 gamma ray-pulsars on the 2PC (Abdo et al. (2013)) and the ~300 gamma-ray pulsars on the 3PC (Smith et al. (2023)). It can be extended to resemble the X-ray regime too, doing it in a majority of the ~40 high-energy pulsars

Geometrical model

- The inclination angle defines the geometry of the emission region





[Viganò & Torres (2019); Íñiguez-Pascual, Torres & Viganò (2024)]

Geometrical model: geometry formulae

- Frenet-Serret equations allow to geometrically describe a curved trajectory with torsion

$$\frac{\mathrm{d}\hat{t}}{\mathrm{d}\lambda} = \frac{1}{r_\mathrm{c}}\hat{n} \qquad \frac{\mathrm{d}\hat{n}}{\mathrm{d}\lambda} = -\frac{1}{r_\mathrm{c}}\hat{t} + \tau\hat{b} \qquad \frac{\mathrm{d}\hat{b}}{\mathrm{d}\lambda} = -\tau\hat{n}$$

- The emission region is build around a centered value of the magnetic colatitude:

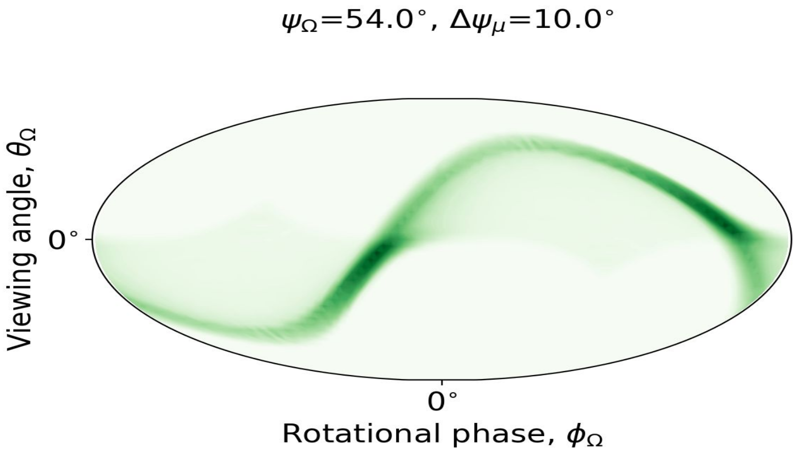
$$\Psi^c_\mu(\xi_\mu, R, \Psi_\Omega) = \frac{\pi}{2} + A(R, \Psi_\Omega)\sin(\xi_\mu - \pi/2) \quad \text{with} \quad A(R, \psi_\Omega) = K\ \Psi_\Omega\ (R/R_\mathrm{lc} - R^0/R_\mathrm{lc})^2$$

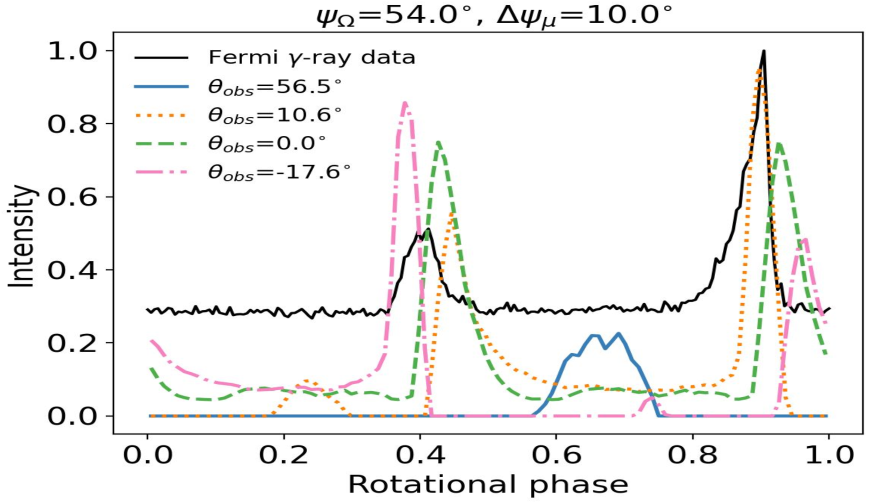
- Synchro-curvature emission maps are generated:

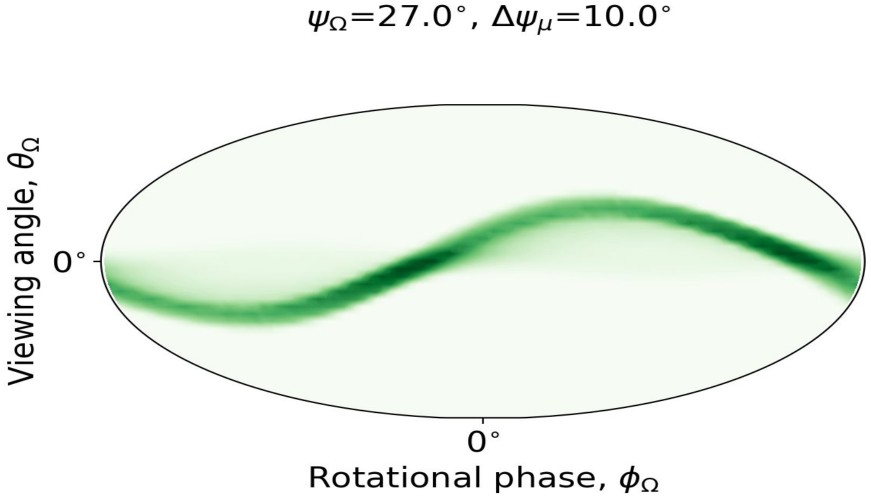
$$M_E(\theta_\mathrm{obs}, \phi_\Omega, E) = \int_{R^0}^{R^0+\Delta R} \int_{\lambda_\mathrm{in}}^{\lambda_\mathrm{out}} \int_0^{2\pi} \int_{\Delta\Psi_\mu} \frac{\mathrm{d}D(\lambda)}{\mathrm{d}\Omega_\Omega} \left\langle \frac{\mathrm{d}P_\mathrm{sc}(\lambda)}{\mathrm{d}E} \right\rangle \frac{\mathrm{d}N(\lambda)}{\mathrm{d}\lambda}\, \mathrm{d}\Psi_\mu\, \mathrm{d}\xi_\mu\, \mathrm{d}\lambda\, \mathrm{d}x_\mathrm{in}$$

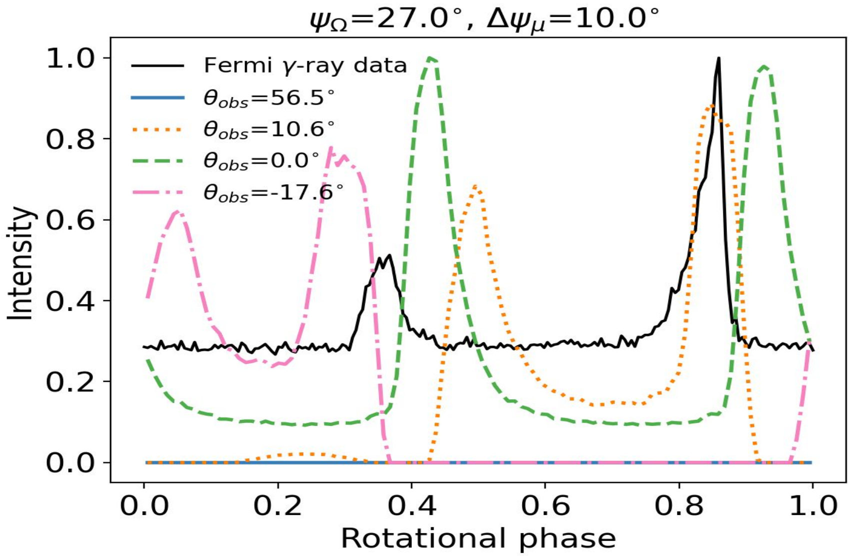
Emission maps and light curves

- We can build emission maps (skymaps), from which light curves are obtained



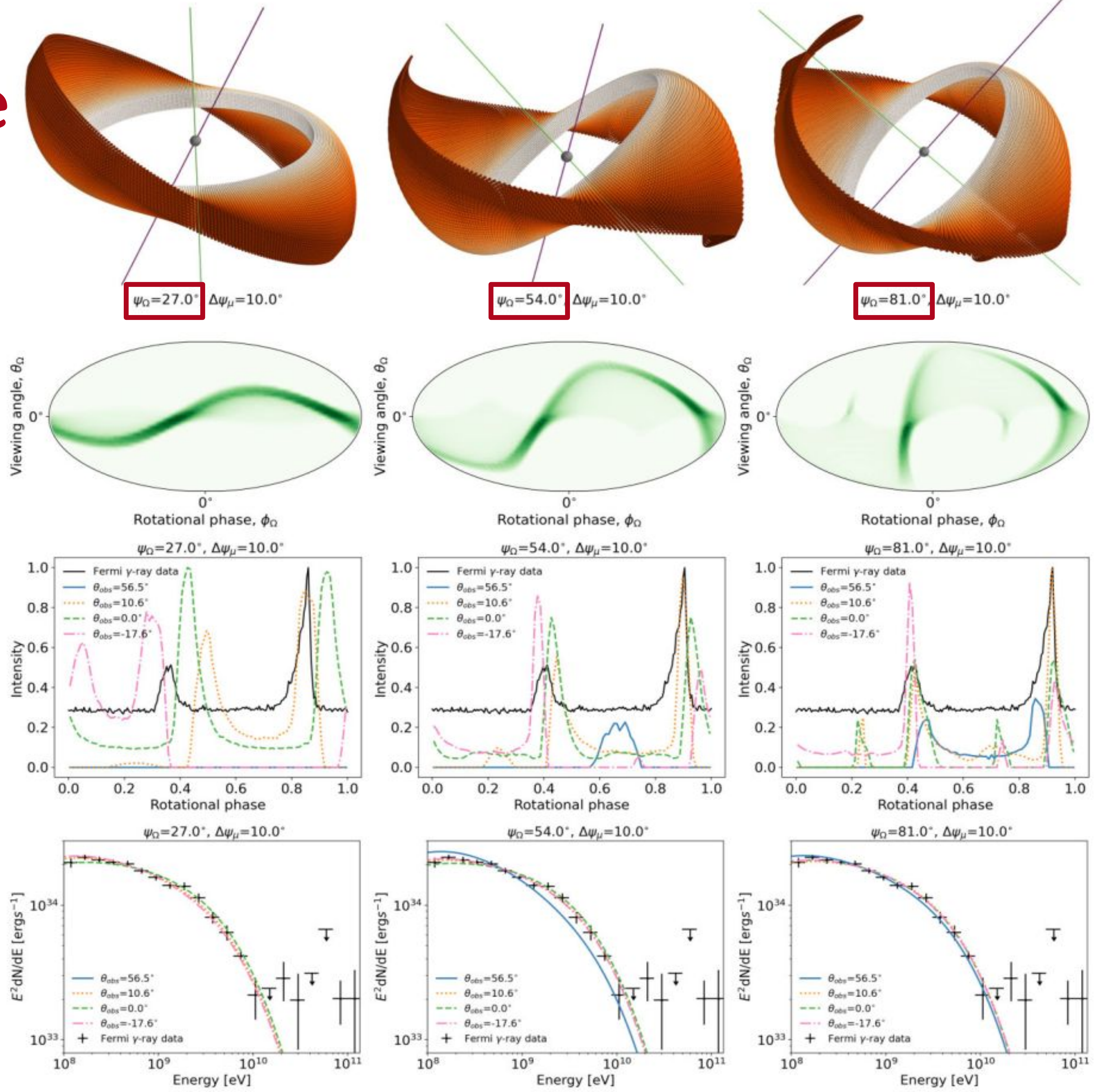






[Íñiguez-Pascual, Torres & Viganò 2024, MNRAS, 530, 1550]

Outputs of the geometrical model



Light curves statistics

[Íñiguez-Pascual, Torres & Viganò 2024, MNRAS, 530, 1550]

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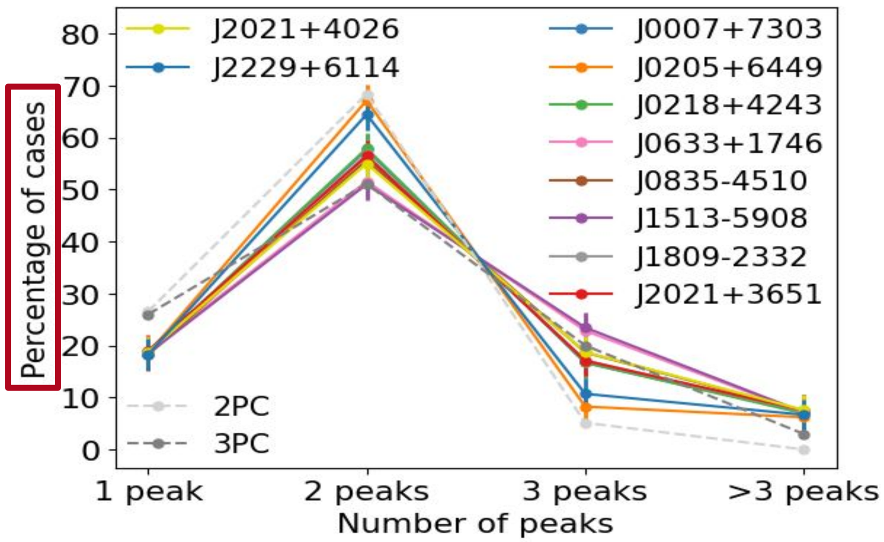

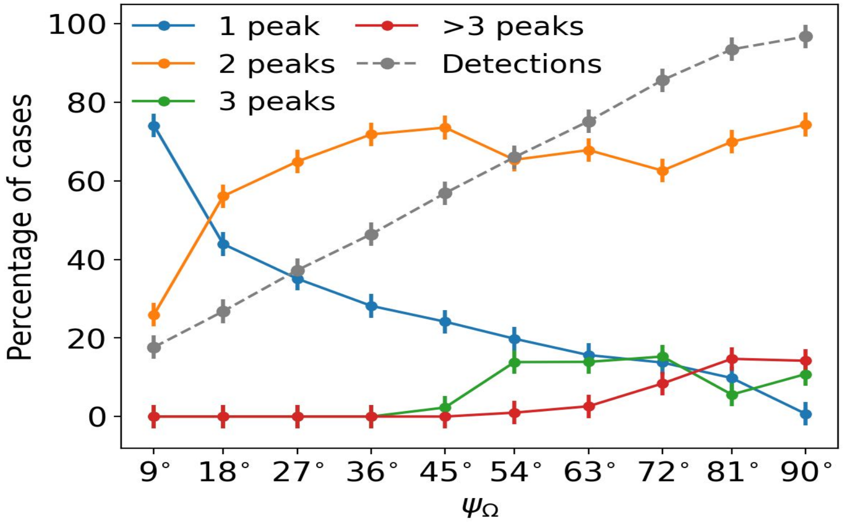

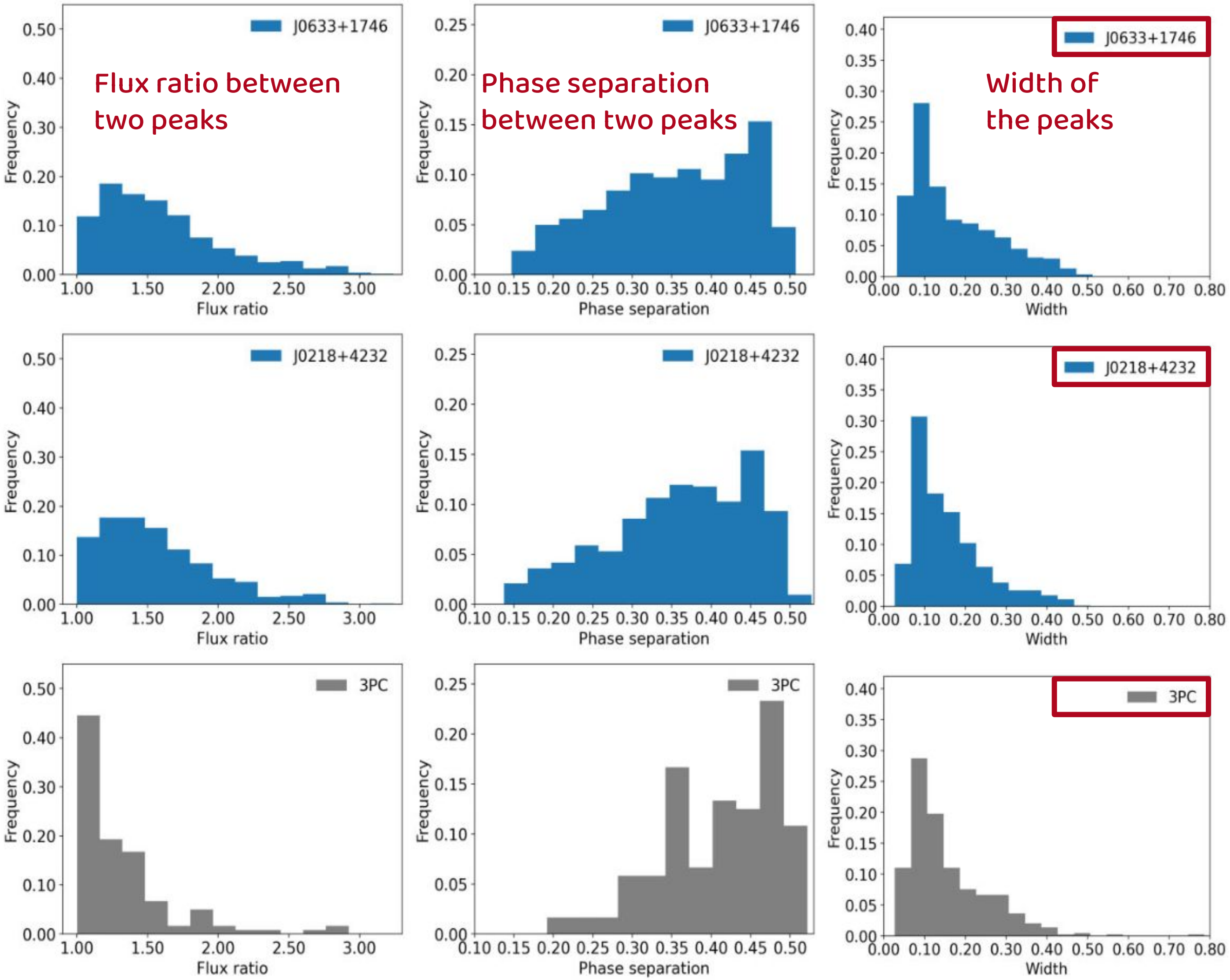

Light curves fitting procedures

- Some considerations have to be made before light curve fitting: equal number of bins, background subtraction and normalization

- Light curve fitting is a non-trivial task, and there is not a perfect metric to be used

- We consider in parallel two methods:
  - A weighted reduced χ2 in time domain

$$\overline{\chi^2_w} = \frac{1}{n-2} \sum_i \frac{(I_i^{obs} - I_i^{syn})^2 \bar{I}_i^{obs}}{(\delta I_i^{obs})^2}$$

  - Euclidean distance in frequency domain

$$||\hat{I}_k|| = K \left|\left| \sum_{j=0}^{n-1} I_i e^{-i2\pi\, k\, j/n} \right|\right| \qquad ED = \sqrt{\sum_k \left(||\hat{I}_k^{obs}|| - ||\hat{I}_k^{syn}|| \right)^2}$$

Light curves fitting

[Íñiguez-Pascual et al. 2025 (submitted)]

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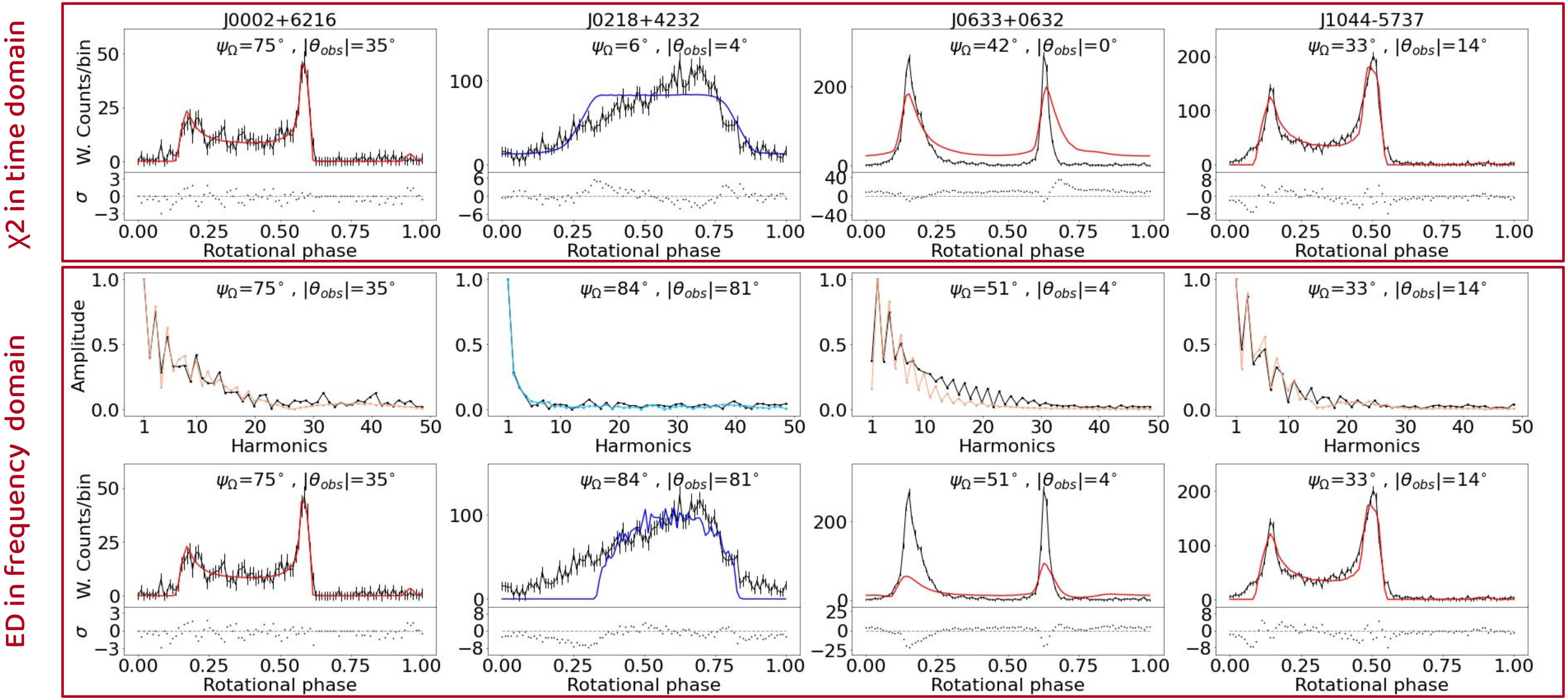
- Fitting synthetic light curves to observational ones, concurrently to the spectral fitting

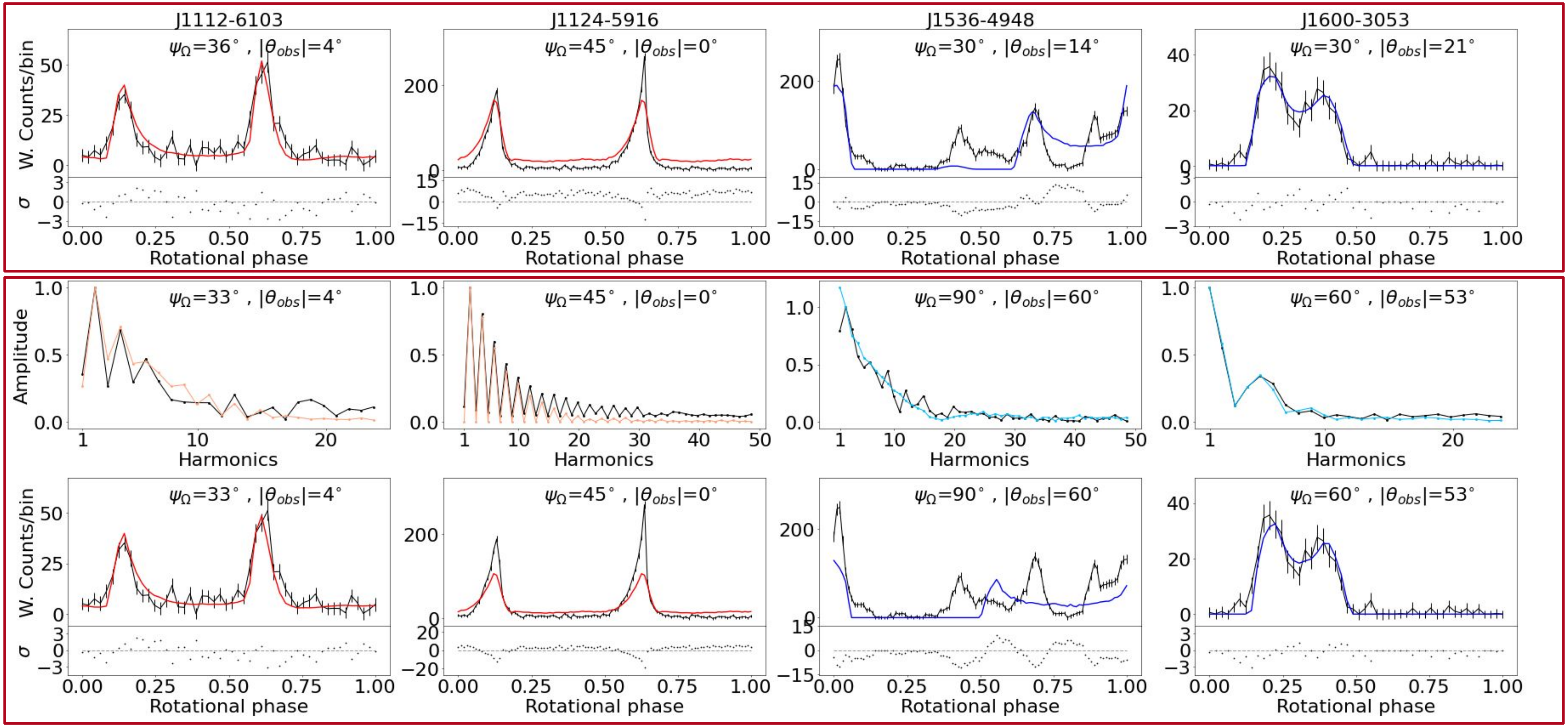


Light curves fitting [Íñiguez-Pascual et al. 2025 (submitted)]

- Fitting synthetic light curves to observational ones, concurrently to the spectral fitting

χ2 in time domain

ED in frequency domain



Light curves fitting

[Íñiguez-Pascual et al. 2025 (submitted)]

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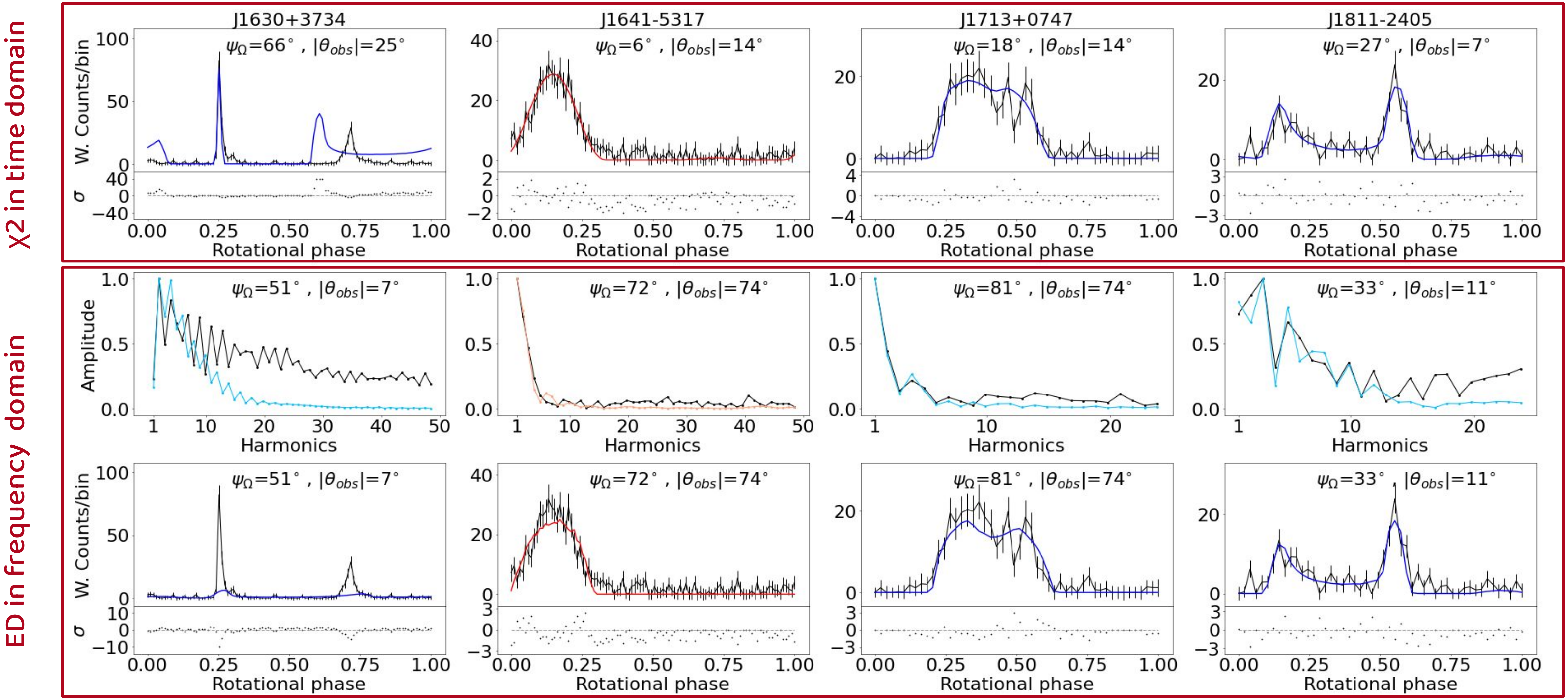

Light curves fitting

[Íñiguez-Pascual et al. 2025 (submitted)]

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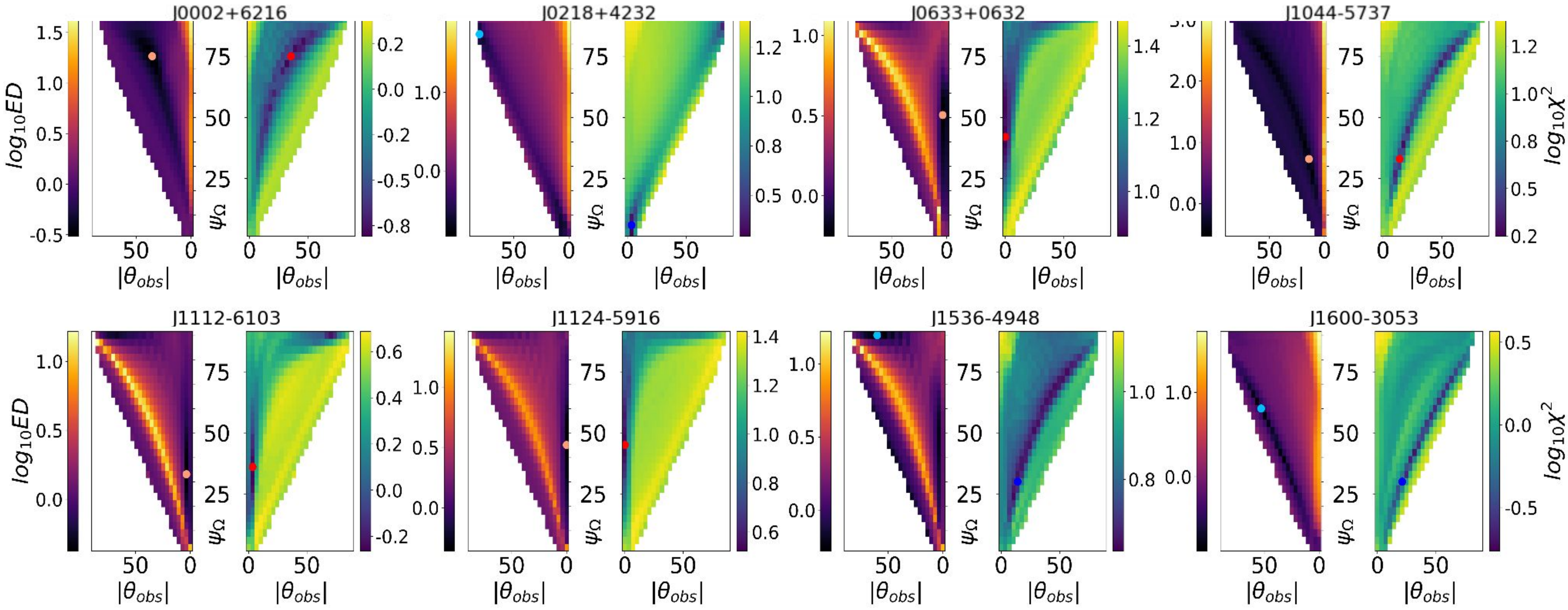

Conclusions

- Concurrently fitting spectra and light curves of gamma-ray pulsars
- With three free spectral parameters, our spectral model is still able to fit the whole high-energy pulsar population
- We properly fit a number of gamma-ray light curves, involving just two free geometrical parameters
- Two differents methods are used for the light curve fitting: a weighted reduced $\chi 2$ in time domain and an euclidean distance in frequency domain, both with qualitatively similar results
- We will improve the model by including more realistic physics while keeping our effective approach

Íñiguez-Pascual D., Viganò D., Torres D. F., 2022, MNRAS, 516, 2475 (2208.05549)

Íñiguez-Pascual D., Torres D. F., Viganò D., 2024, MNRAS, 530, 1550 (2404.01926)

Íñiguez-Pascual D., Torres D. F., Viganò D., 2025, submitted (2504.01892)

Thank you

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Spectral model: particle dynamics formulae

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- The equation of motion of charged particles balances electric acceleration and synchro-curvature losses

$$\frac{d\vec{p}}{dt} = ZeE_{\|}\hat{b} - (P_{sc}/v)\hat{p} \qquad \vec{p} = \Gamma m \vec{v}$$

[Viganò et al. (2015)]

- We solve it numerically, considering separately the components parallel and perpendicular to the trajectory

$$\frac{d(p\cos\alpha)}{dt} = ZeE_{\|} - \frac{P_{sc}}{v}\cos\alpha \qquad \frac{d(p\sin\alpha)}{dt} = -\frac{P_{sc}}{v}\sin\alpha$$

- Local magnetic field strength and curvature radius are parametrize in an effective way:

$$B = B_{\star}\left(\frac{R_{\star}}{x}\right)^{b} \qquad r_c = R_{lc}\left(\frac{x}{R_{lc}}\right)^{\eta}$$

Spectral model: synchro-curvature formulae

- Single-particle synchro-curvature power spectra:

$$\frac{dP_{sc}}{dE} = \frac{\sqrt{3}(Ze)^2\Gamma y}{4\pi\hbar r_{eff}}\left[(1+z)F(y) - (1-z)K_{2/3}(y)\right]$$

- Synchro-curvature power: $P_{sc} = \frac{2(Ze)^2\Gamma^4 c}{3r_c^2} g_r$

[Cheng & Zhang (1996), Viganò et al. (2015)]

$$r_{eff} = \frac{r_c}{\cos^2\alpha}\left(1+\xi+\frac{r_{gyr}}{r_c}\right)^{-1}$$

$$z = (Q_2 r_{eff})^{-2}$$

$$y = \frac{E}{E_c}$$

$$Q_2^2 = \frac{\cos^4\alpha}{r_c^2}\left[1+3\xi+\xi^2+\frac{r_{gyr}}{r_c}\right]$$

$$E_c = \frac{3}{2}\hbar c Q_2 \Gamma^3$$

$$F(y) = \int_y^\infty K_{5/3}(y')dy'$$

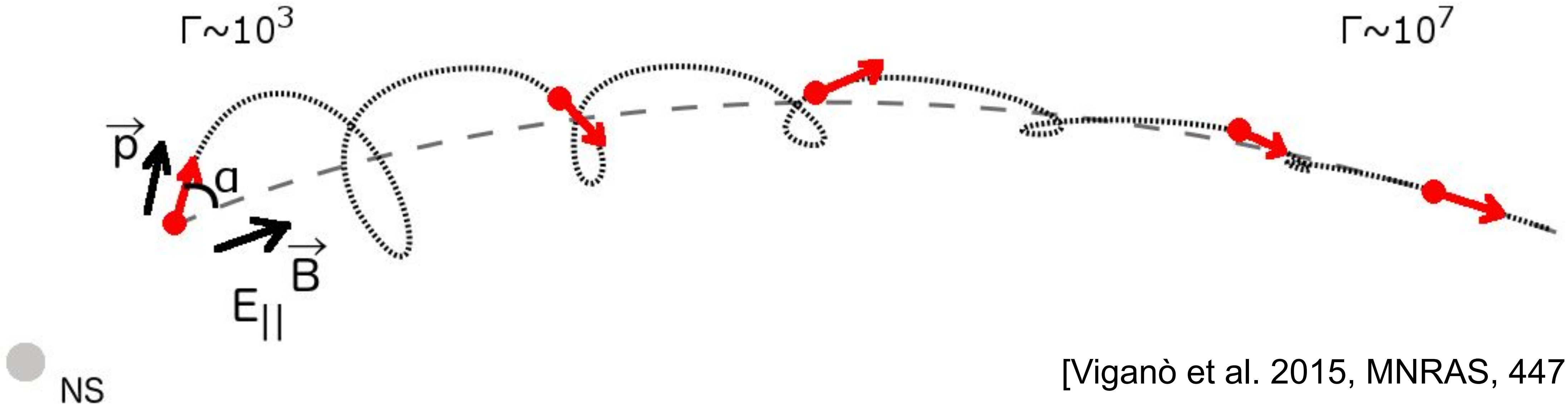
$$\xi = \frac{r_c}{r_{gyr}}\frac{\sin^2\alpha}{\cos^2\alpha}$$

Back up slide: particle dynamics

- Power of synchrotron and curvature radiations depend on the Lorentz factor Γ of particles differently

$$P_{syn} = \frac{2}{3}\frac{(Ze)^4 B^2 (\Gamma^2 - 1)\sin^2\alpha}{m^2 c^3}$$

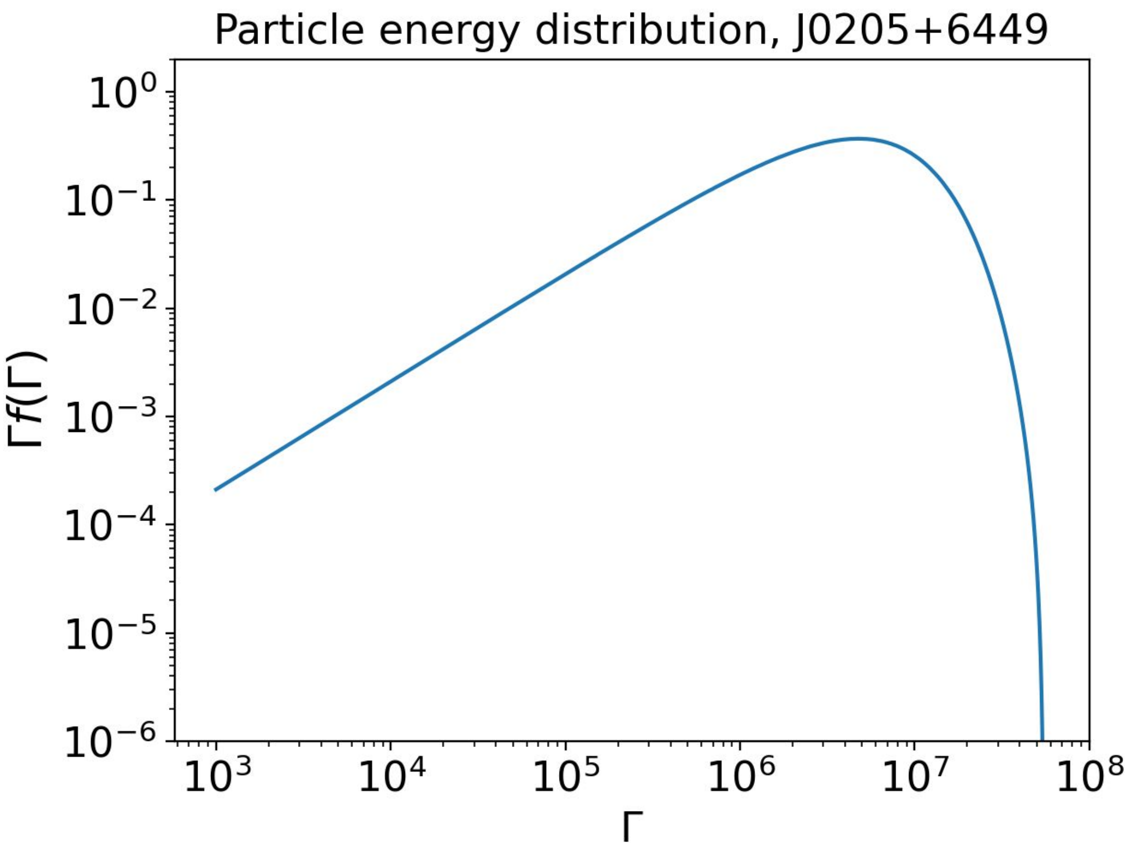
$$P_c = \frac{2}{3}\frac{(Ze)^2 c\Gamma^4}{r_c^2}$$

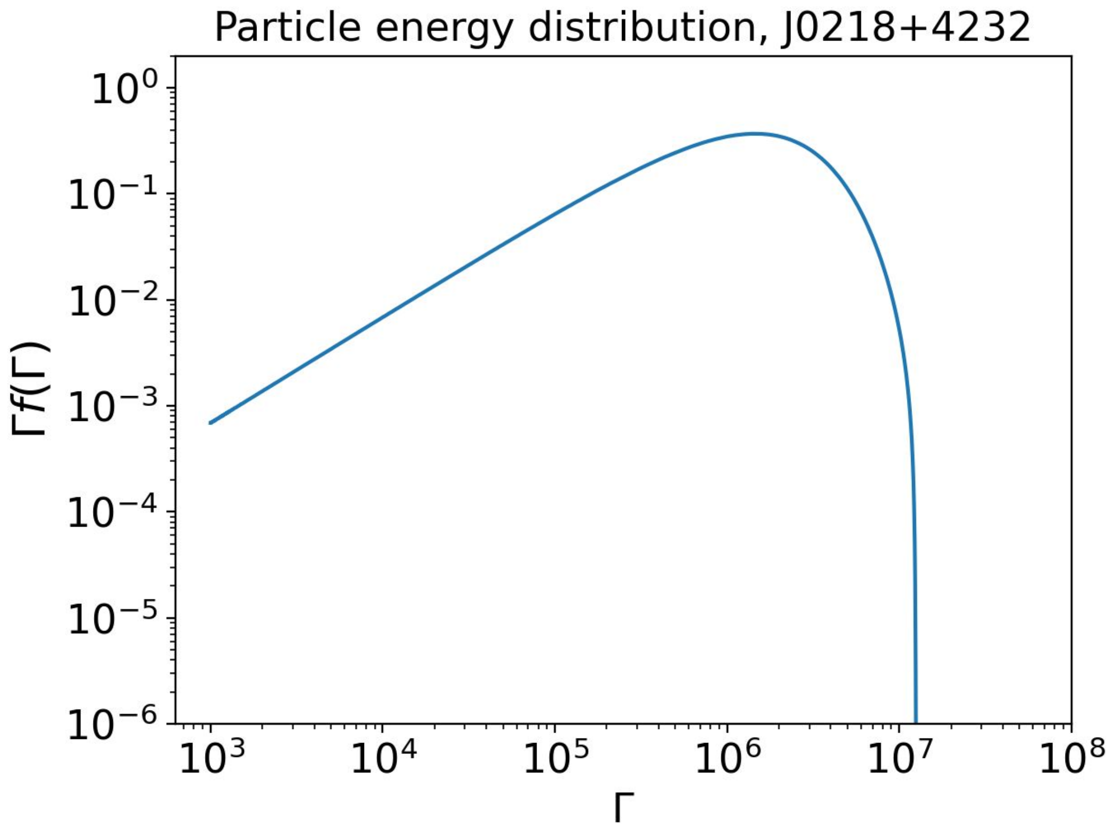


[Viganò et al. 2015, MNRAS, 447, 1164]

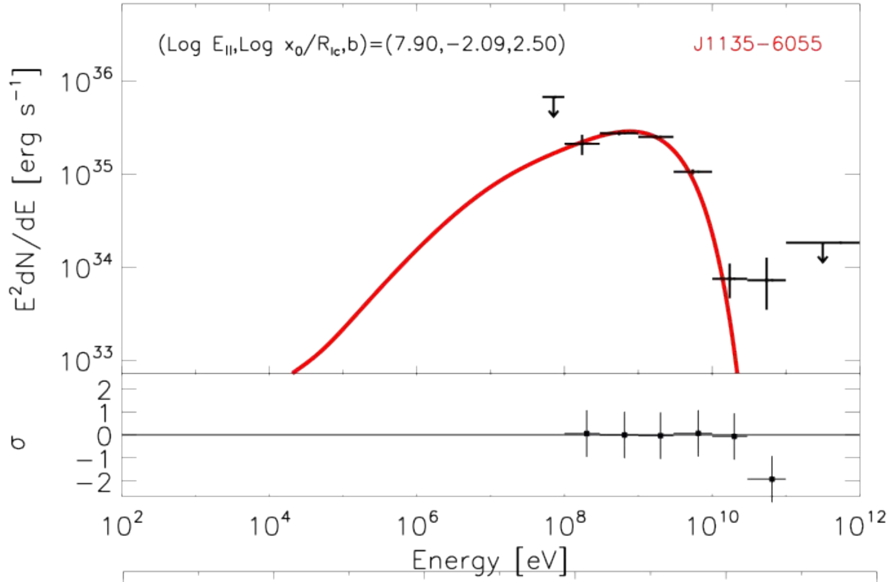
Back up slide: particle energy distribution

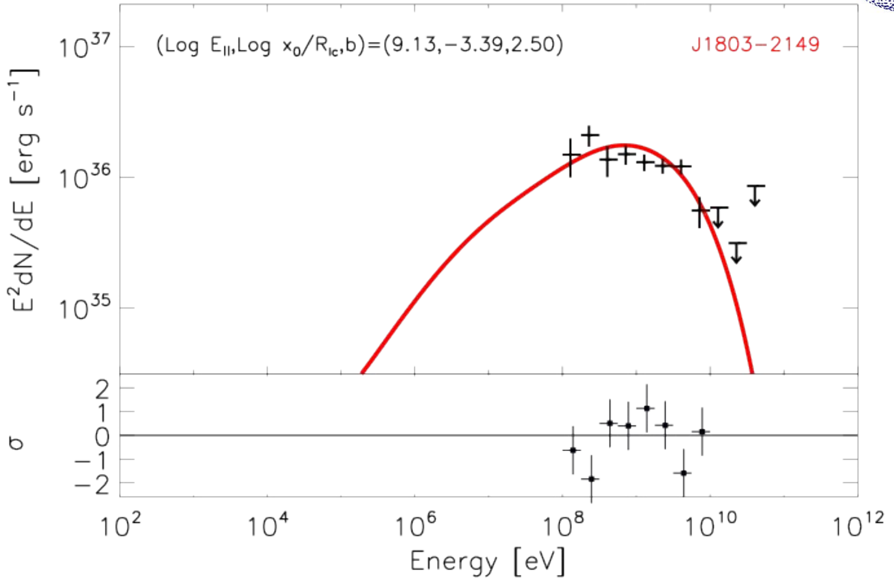
- Lorentz factors Γ typically range from 10^3 to 10^7.

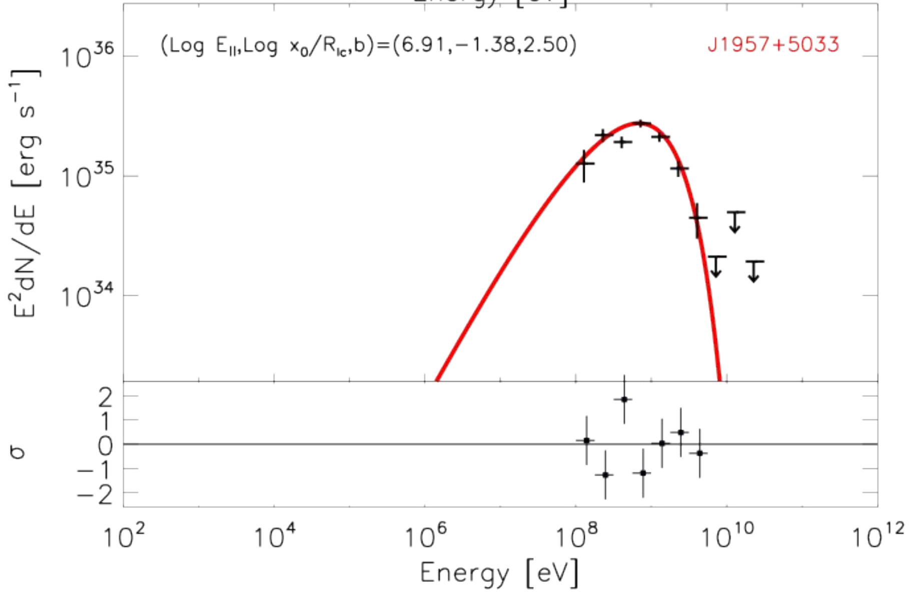


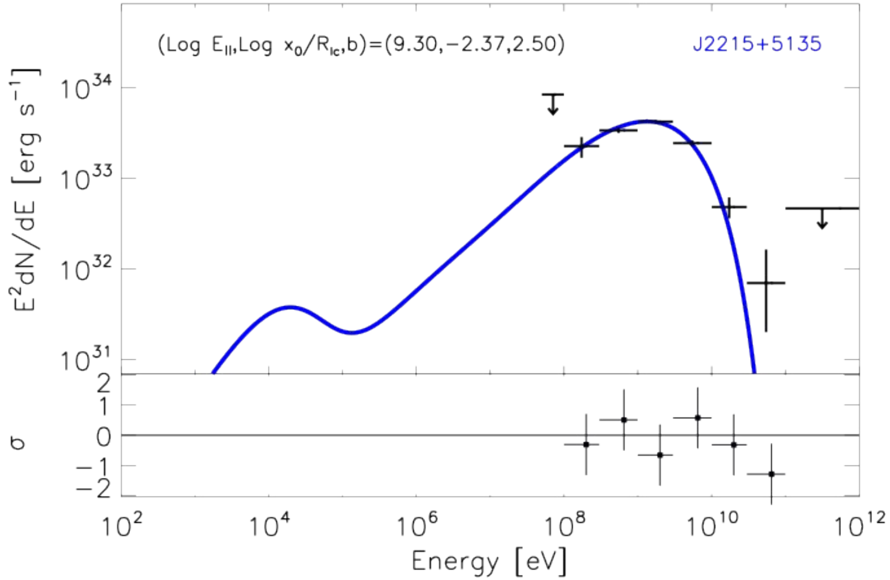


Back up slide: more spectral fits

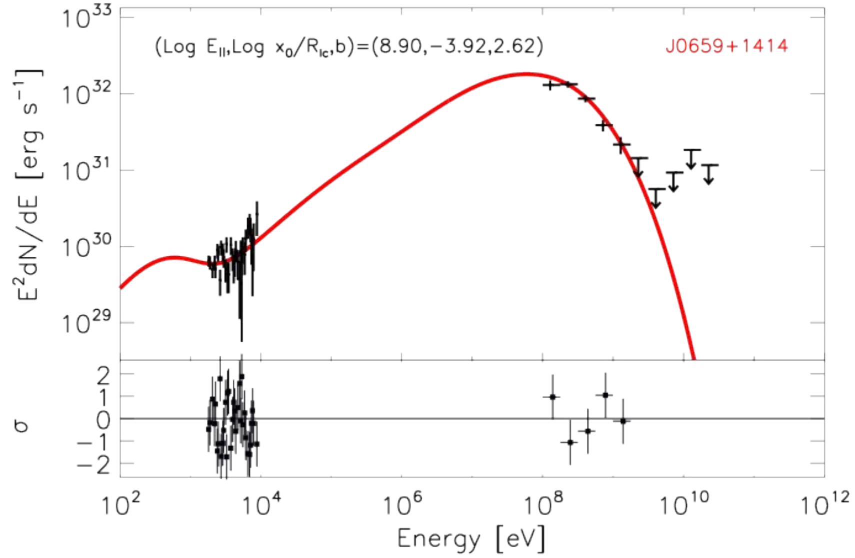


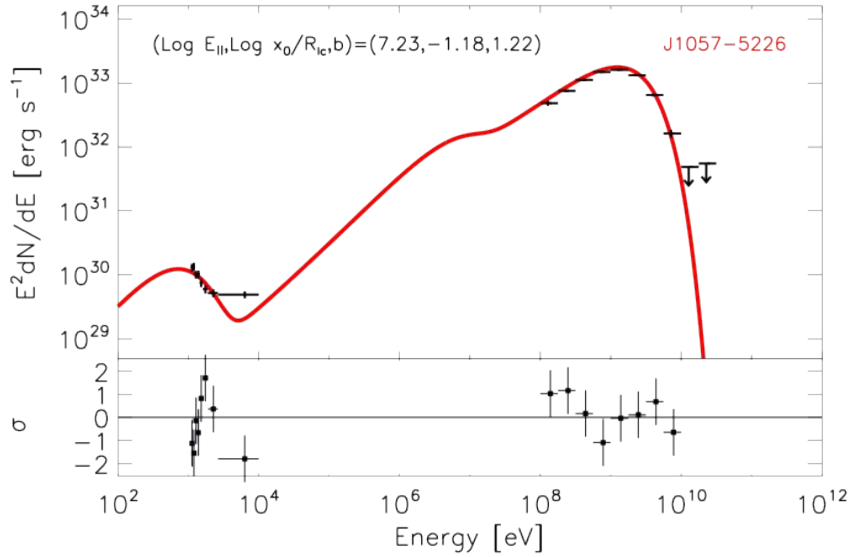


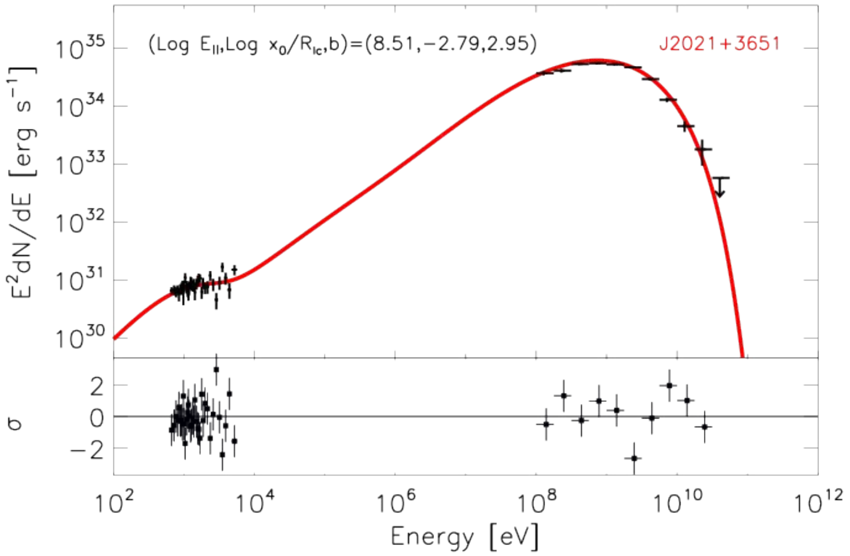


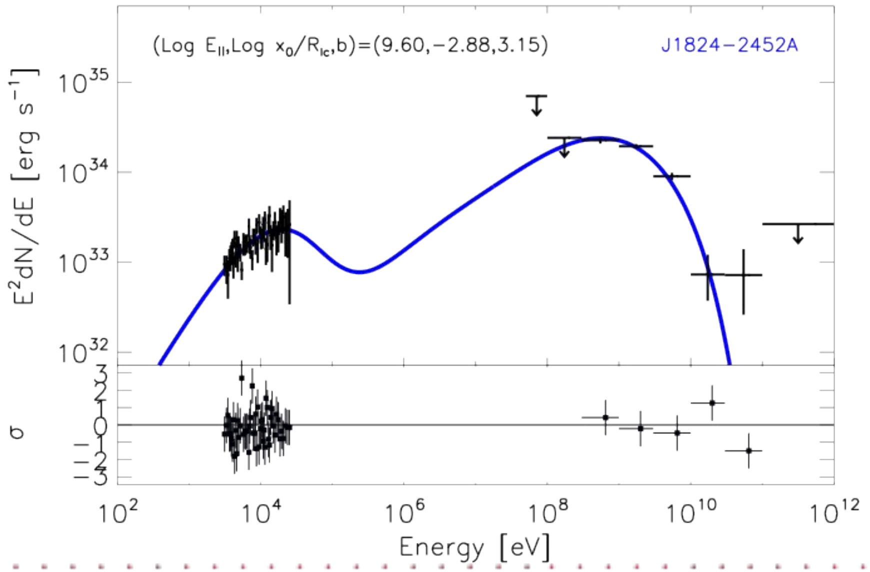


Back up slide: more spectral fits

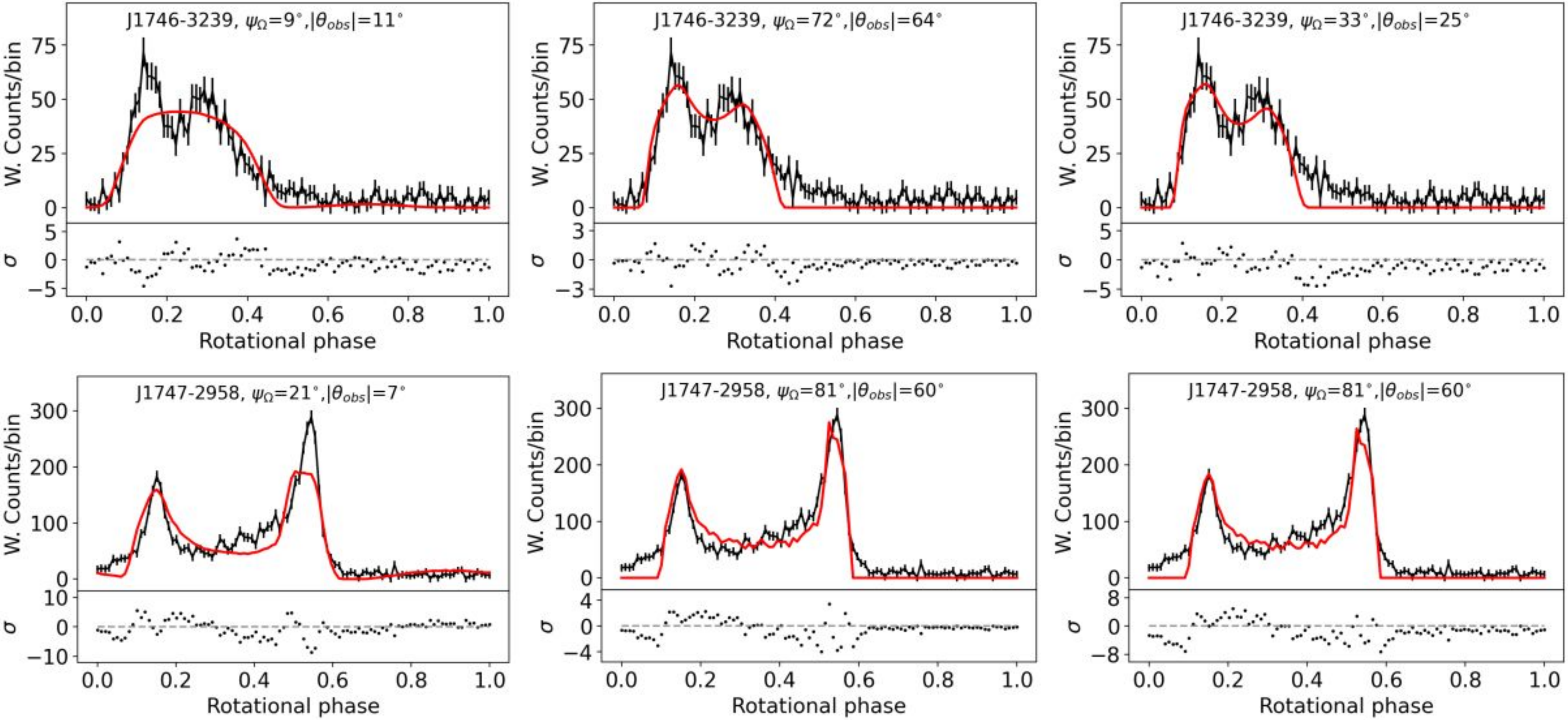








Back up slide: caveats



Back up slide: filtering out Fourier

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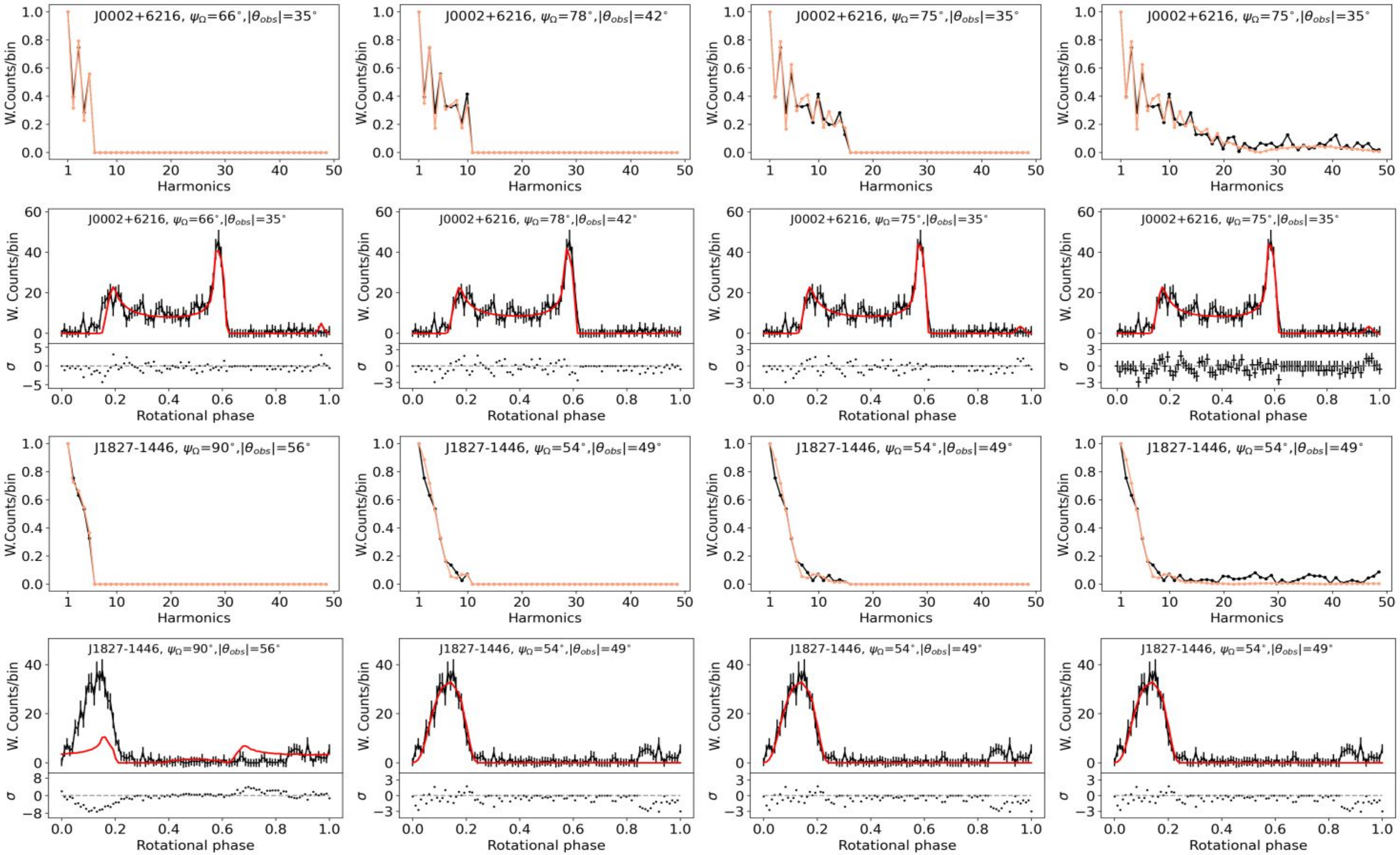
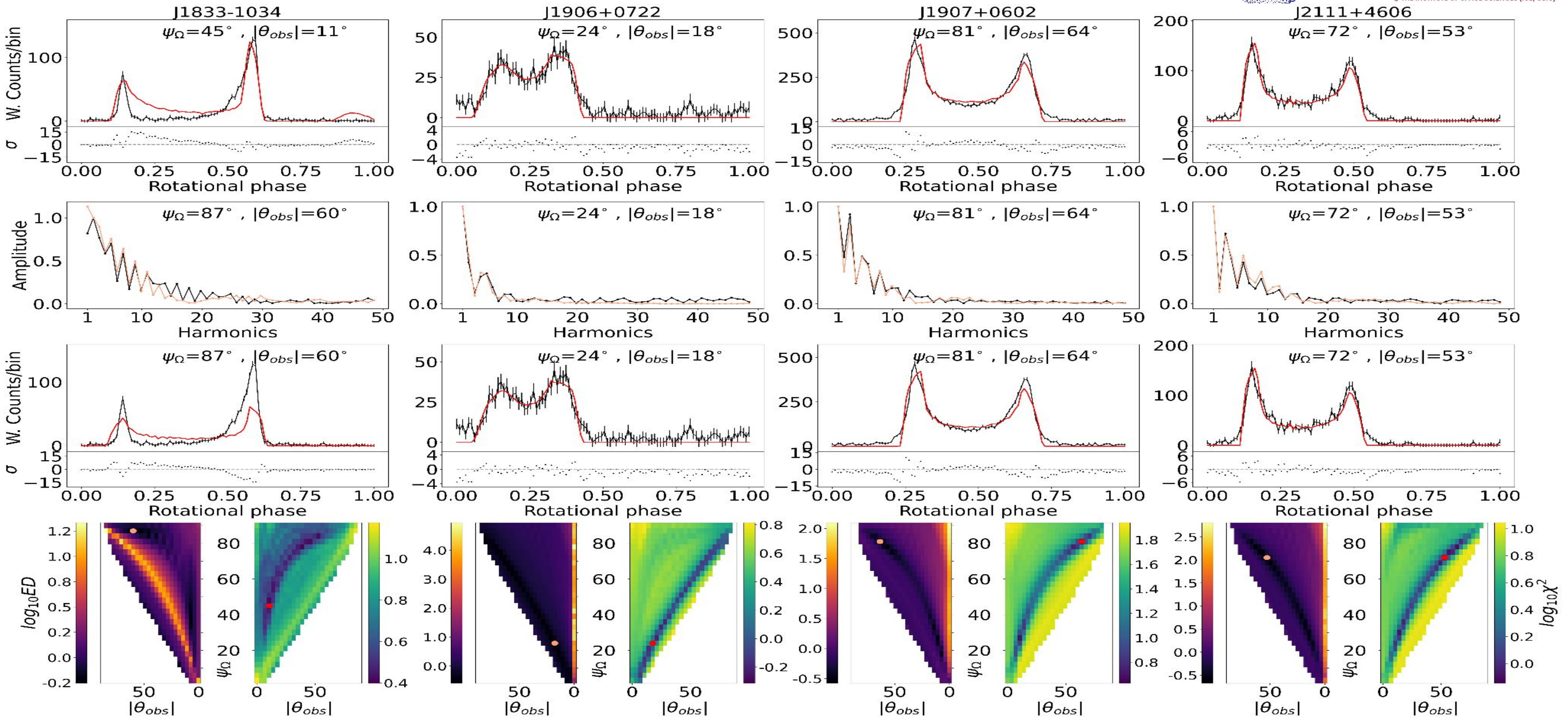


Back up slide: more light curve fits



Back up slide: Degeneracy of geometrical parameters