Force-free-PIC numerical approach to model pulsar magnetospheres

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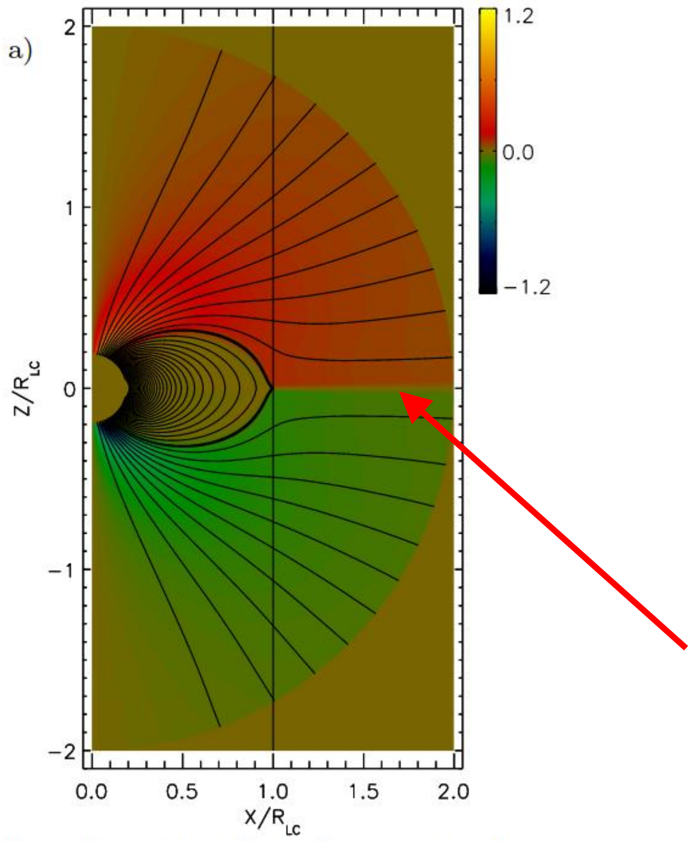
Two global approaches: Force-free & PIC simulations

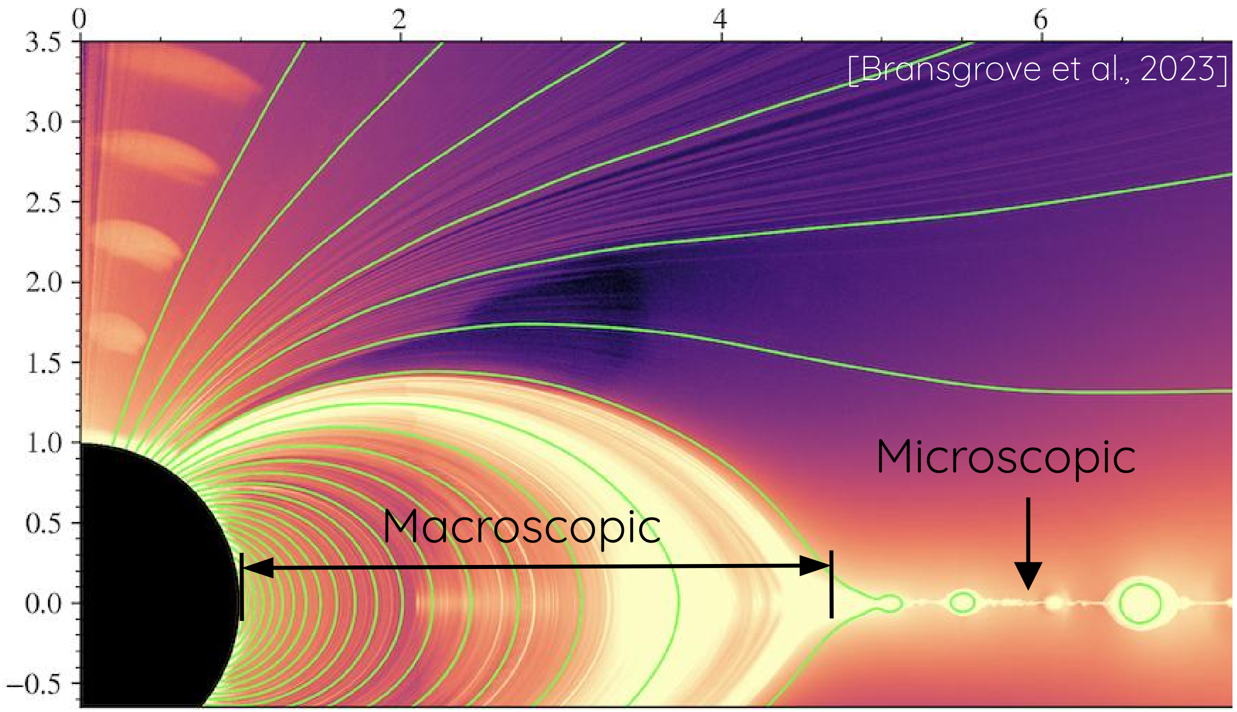


[Spitkovsky 2006, Pétri 2012, Kalapotharakos et al. 2012, …]

- Fluid approach
- Global magnetosphere structure
- Computationally cheap

Limitations:
- Numerically unstable: system breaks
- Cannot capture dissipation by construction
- No information on the particles

Two global approaches: Force-free & PIC simulations



Rescaling procedure: reduce magnetic field strength + neutron star size (unrealistic)

$r_\star = 10^2$ cm $\quad B_\star = 10^5$ G

- First-principle simulation
- Capture dissipation
- Produce synthetics
- Computationally expansive
- Must resolve microphysics
- Unrealistic scale separation

Target of the simulation: weakest Fermi MSP

- Pulsar radius:

$$r_\star = 10^6 \mathrm{cm}$$

- Spin period:

$$P = 1\mathrm{ms}$$

- Aligned dipole:

$\chi = 0°$ ⟶ 2D axisymmetric

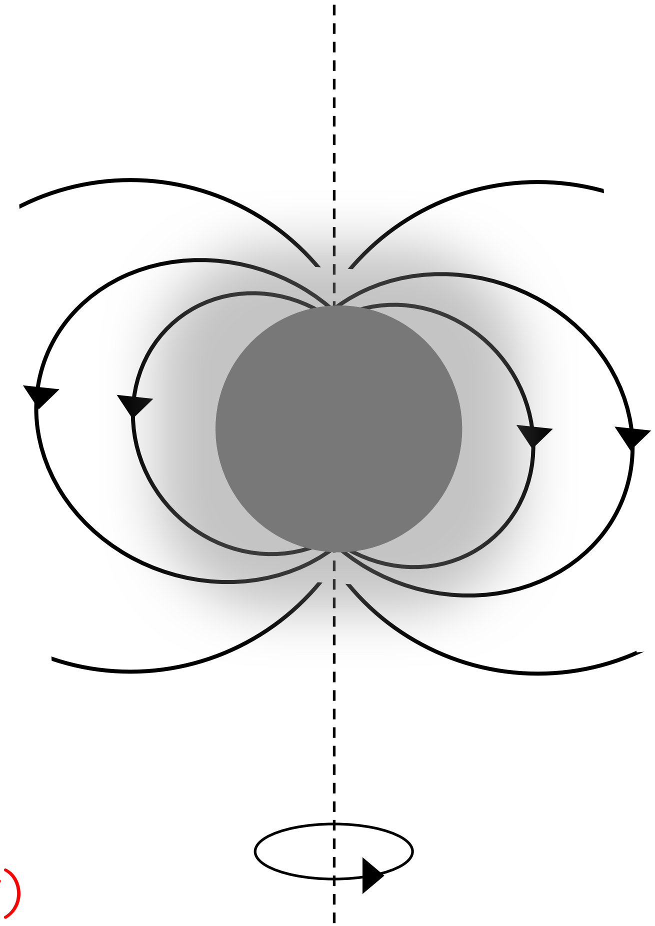
- Surface magnetic field:

$$B_\star = 10^7 \mathrm{G}$$

- Spindown power:

$$\mathrm{L}_0 = 4.8 \times 10^{33}\ \mathrm{erg/s}$$

- **Focus on gamma-ray emission** (current sheet)



Different scales for MSP

Spatial scales

Macroscopic

- star radius $r_\star = 10^6$ cm
- light-cylinder $R_{\rm LC} = 5 r_\star$

Microscopic

- skindepth (surface)
  $d_{\rm e}^\star \approx 2 \times 10^{-6} r_\star$
- secondary skindepth
  $d_{\rm e}^{\star,\rm s} \approx 10^{-3} r_\star$
- current sheet thickness
  $\delta \sim d_{\rm e}^{\rm LC} \approx 2 \times 10^{-2} r_\star$

Lorentz factor scales

Full potential drop (polar cap)

$$\gamma_{\rm pc} \approx 2.6 \times 10^8$$

At light-cylinder

$$\gamma_{\rm LC} \approx 1.3 \times 10^6$$

Pair production threshold

$$\gamma_{\rm th} \approx 10^6$$

Secondary

$$\gamma_{\rm s} \approx 2.2 \times 10^5$$

Radiation-reaction

$$\gamma_{\rm rad} \approx 3 \times 10^4 \qquad \gamma_{\rm rad}^{\rm LC} \approx 3.3 \times 10^5$$

Different scales for MSP

System scales | Lorentz factor scales

Macrosco…

Microscop…

Required scale separation

Spatial

$$\frac{d_{\rm e}^{\rm s}}{r_\star} \sim 10^{-3} \ll \frac{\delta}{r_\star} \sim 2.5 \times 10^{-2} \ll 1 < \frac{R_{\rm LC}}{r_\star} = 5$$

Energy

$$\frac{\gamma_{\rm s}}{\gamma_{\rm pc}} \sim 8.5 \times 10^{-4} < \frac{\gamma_{\rm rad}^{\rm LC}}{\gamma_{\rm pc}} \sim 10^{-3} < \frac{\gamma_{\rm th}}{\gamma_{\rm pc}} \sim 4 \times 10^{-3} \ll 1$$

$d_{\rm e}^{\rm s} \approx 10^{-3} r_\star$

current sheet thickness

$\delta \sim d_{\rm e}^{\rm LC} \approx 2 \times 10^{-2} r_\star$

Radiation-reaction

$\gamma_{\rm rad} \approx 3 \times 10^4$ $\quad \gamma_{\rm rad}^{\rm LC} \approx 3.3 \times 10^5$

Bridging the gap between simulations and observations

Can we still perform a simulation with these MSP constraints?

Short answer: not with the current tools and without sacrifices

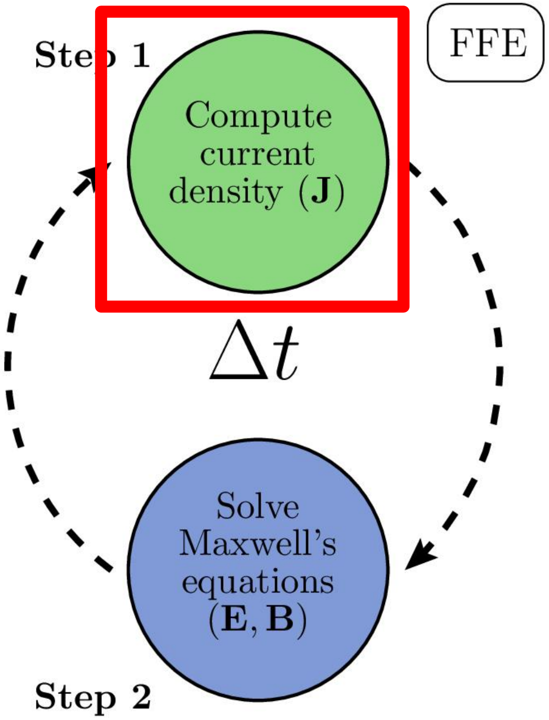
Possible solution: Build a new numerical method using both force-free and PIC in a unique numerical framework

Next questions: Why? How? (Coupling procedure, criterion)

Why coupling Force-free and PIC?

Method	FFE	PIC
Global scales	✓	✗
Microphysics	✗	✓
Particle acceleration	✗	✓
Energy dissipation	✗	✓
Computing time	✓	✗

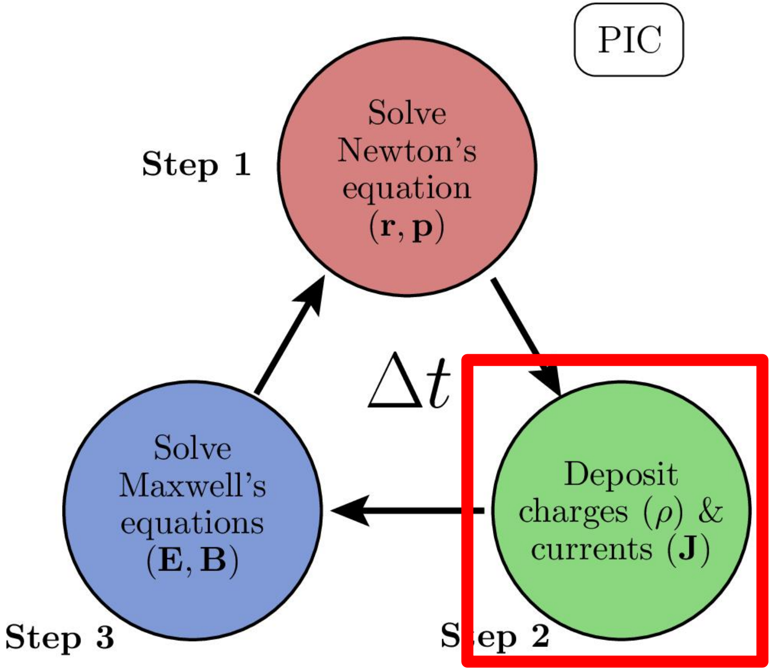
Coupling Force-free and PIC algorithms



Maxwell's equations

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c\nabla \times \mathbf{B} - 4\pi \mathbf{J}$$



$\mathbf{J}_{\mathrm{FFE}}$ → $\mathbf{J}\left(\mathbf{J}_{\mathrm{PIC}}, \mathbf{J}_{\mathrm{FFE}}\right)$ ← $\mathbf{J}_{\mathrm{PIC}}$

Hybrid criterion

- Domain separation criterion:

  Magnetic flux function $\Psi = \iint \mathbf{B} \cdot \mathrm{d}\mathbf{S}$

  ➜ Isocontours of Ψ = magnetic field lines
- Separatrix ➜ γ-rays ➜ PIC description

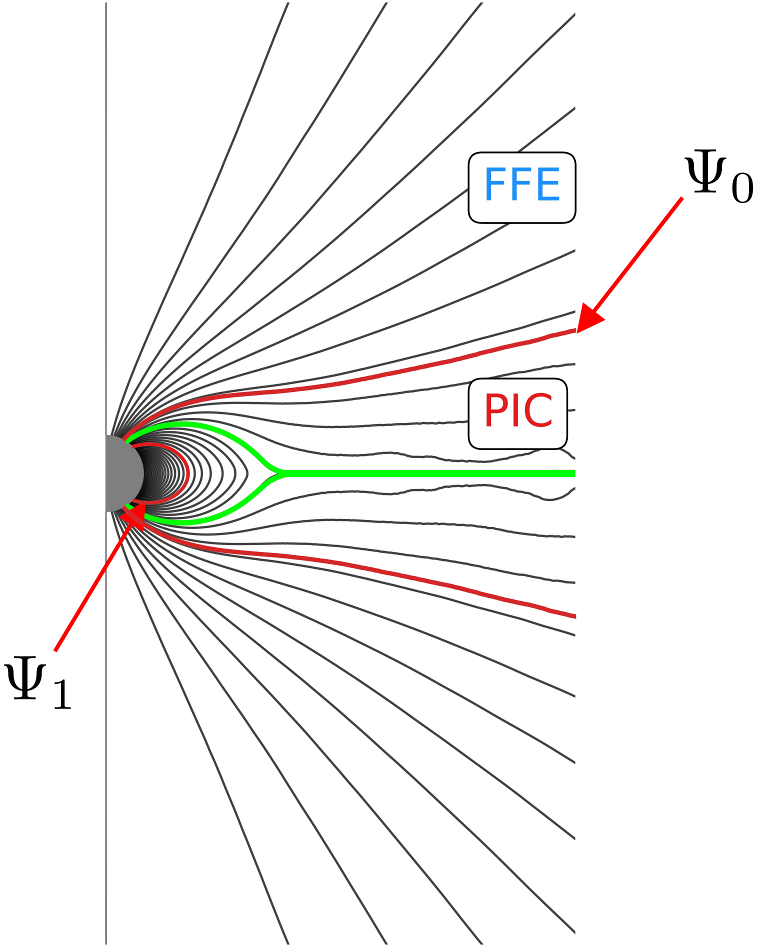
If $\Psi \in [\Psi_0, \Psi_1]$:

↳ PIC

Else:

↳ Force-free

Consequences: sacrifice the polar caps



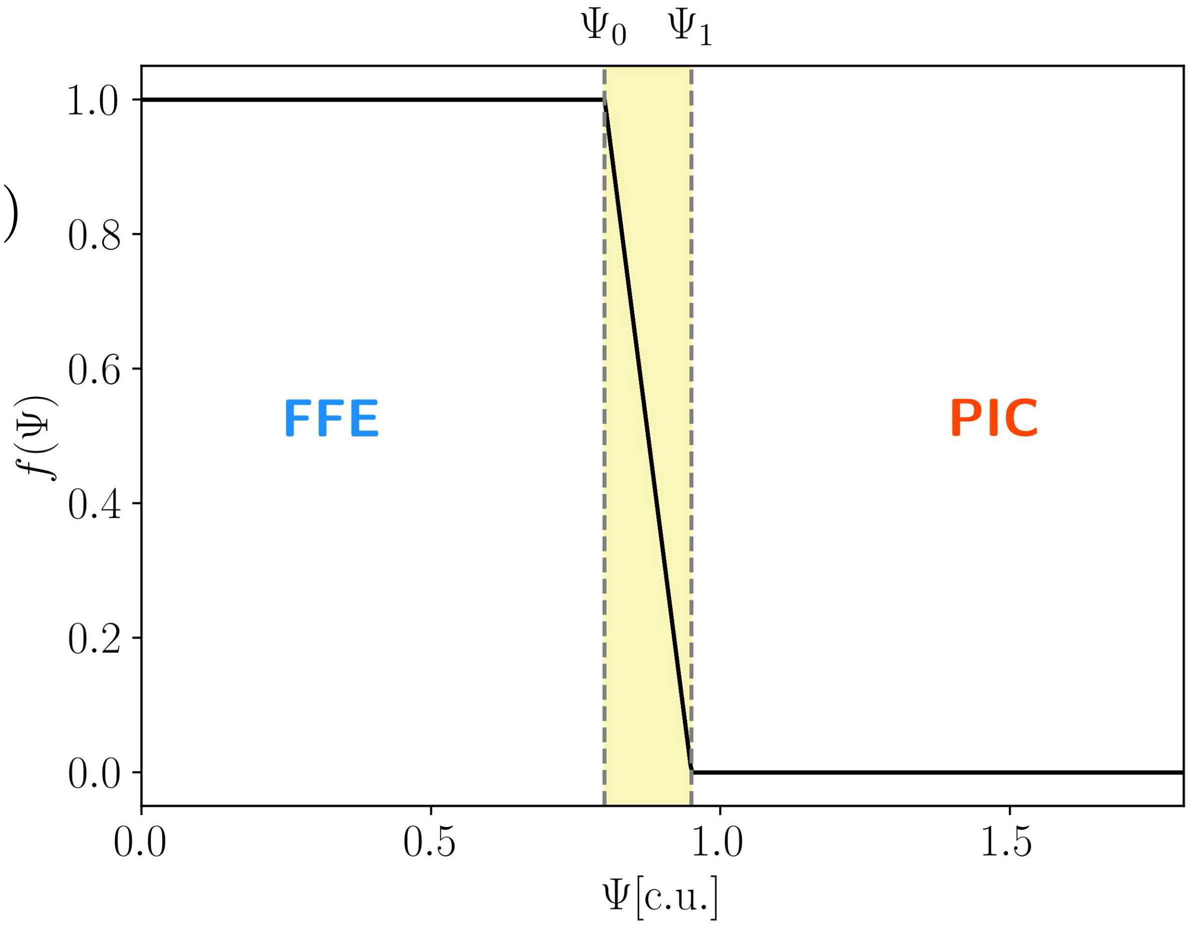
Blending function

- Coupled current density:

$$\mathbf{J} = \mathbf{J}_{\mathrm{PIC}}\,(1 - f(\Psi)) + \mathbf{J}_{\mathrm{FFE}}\,f(\Psi)$$

- $f(\Psi)$ = blending function:
  ➜ linear interpolation

- Finite thickness



Numerical setup

Code: Zeltron [Cerutti et al. 2013]

Simulation cost: 0.5M CPUh

Simulation box: $r = [r_\star, 3R_{\rm LC}]$

$\theta = [0, \pi]$

Resolution: (r,θ)=(8192,8192)

Smallest cell: $\Delta r = 3.4 \times 10^2 \text{ cm} < d_{\rm e}^{\rm s}$

Initial field: aligned dipole with $B_\star = 10^7 \rm G$

Spin period: $\rm P = 1ms$

Injection: surface (e- / p)

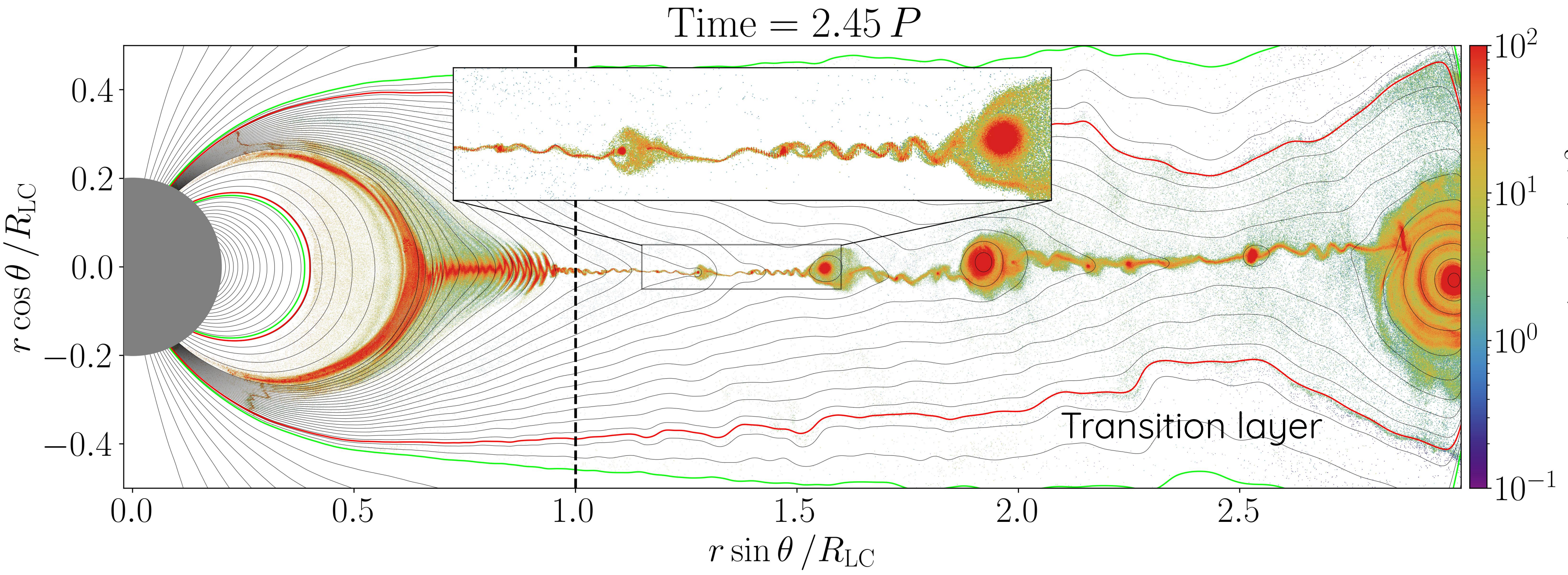
Mass ratio: $\rm m_i / m_e = 1836$

Pair production: energy threshold

Radiation: Curvature + synchrotron

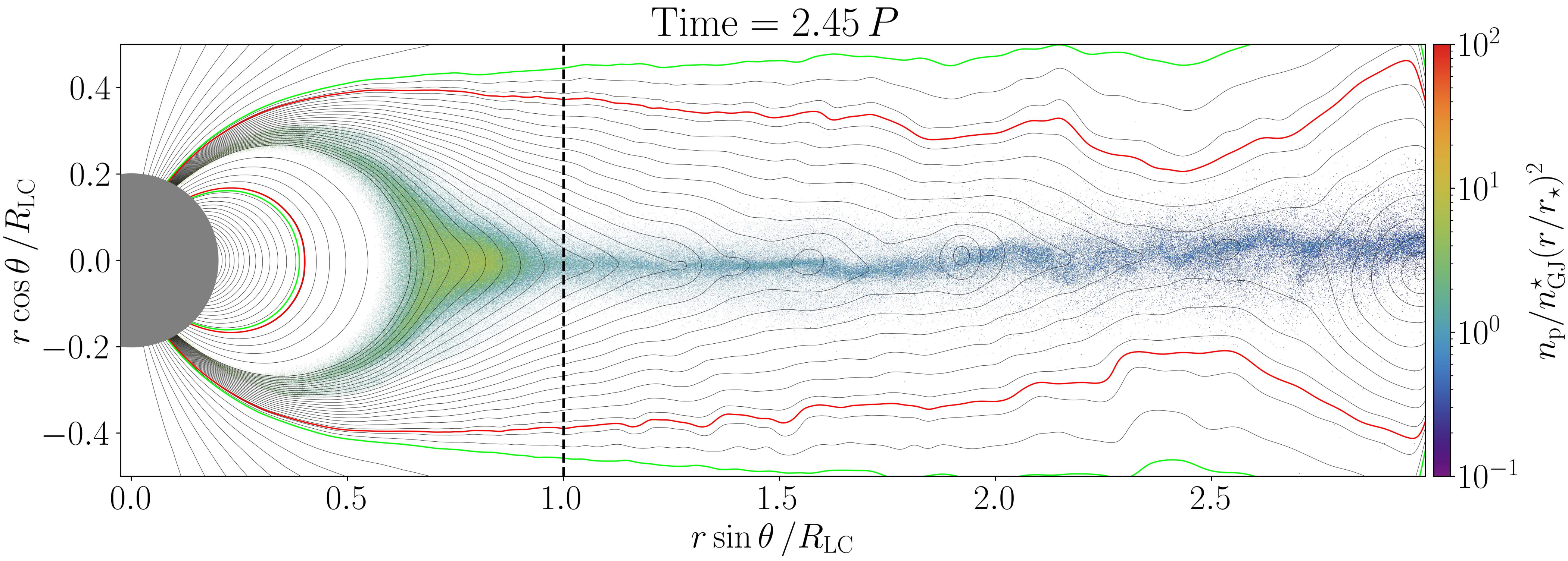
No rescaling procedure in this simulation

Pairs density



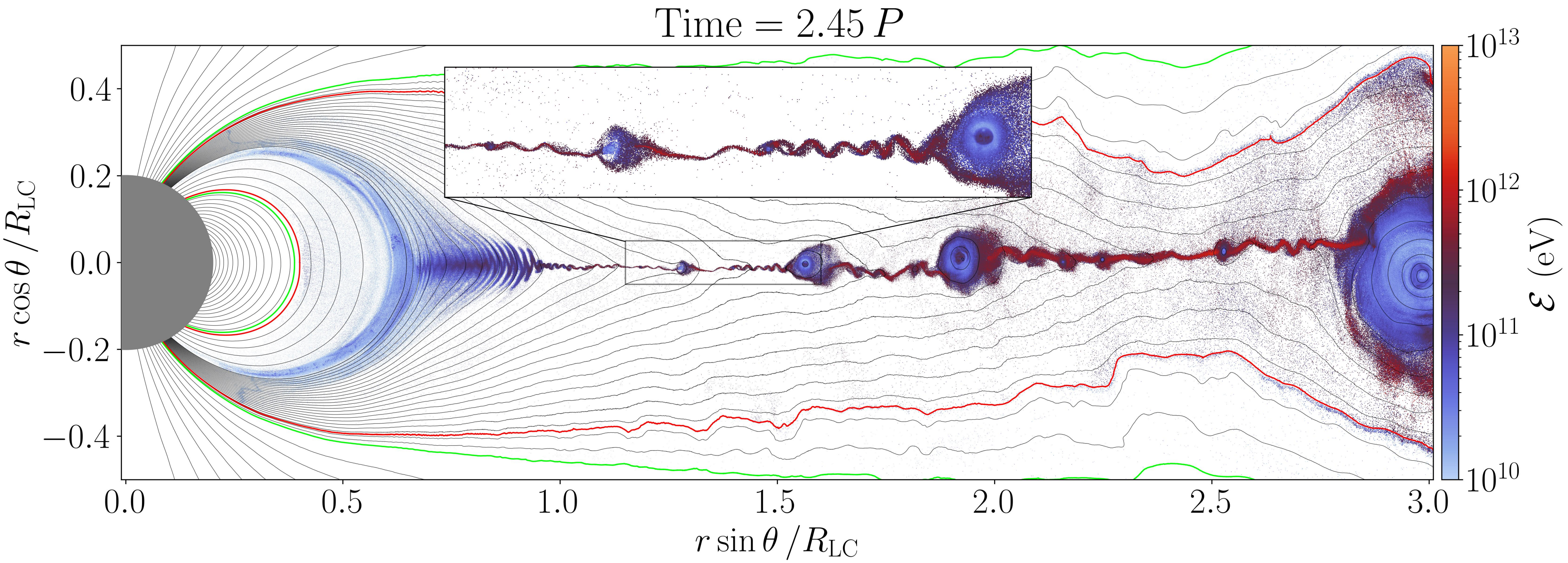
- Current sheet thin & dynamic
- Magnetic reconnection
- Dense plasmoids

Protons density



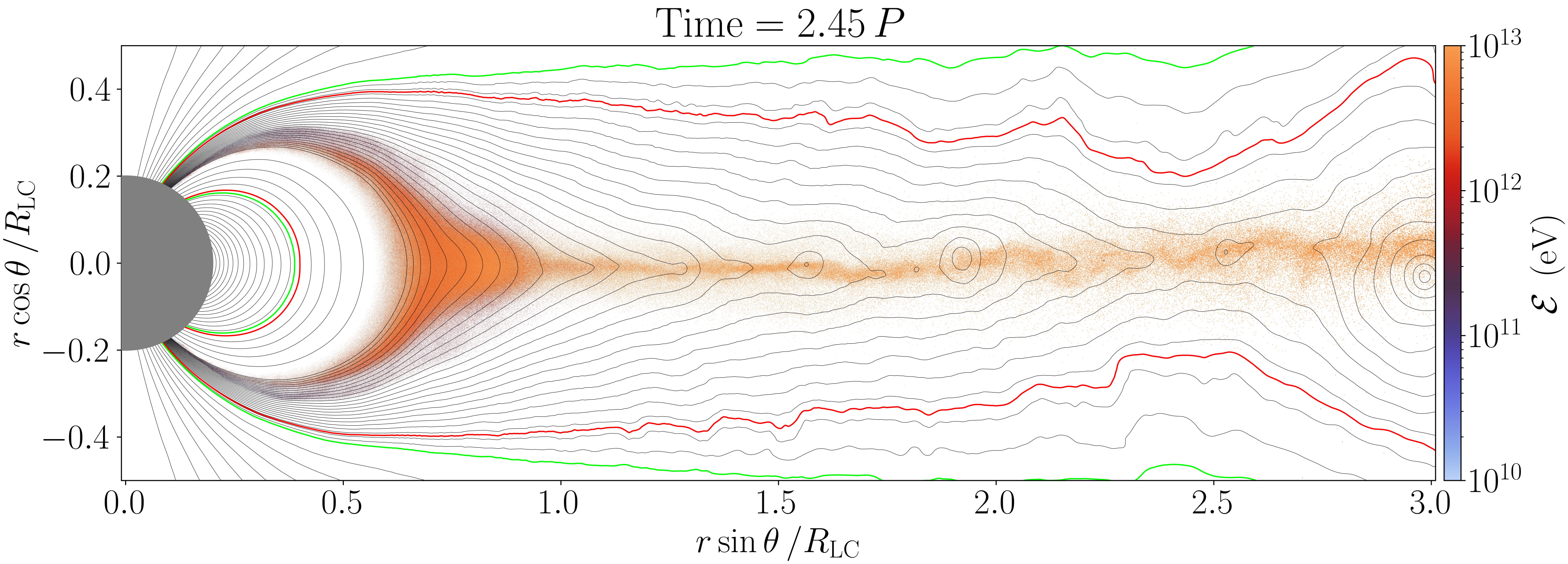
- Inside the current layer
- Low density
- Not impacted by plasmoids

Pairs energy



- X-points = acceleration site
- Plasmoids = catastrophic cooling

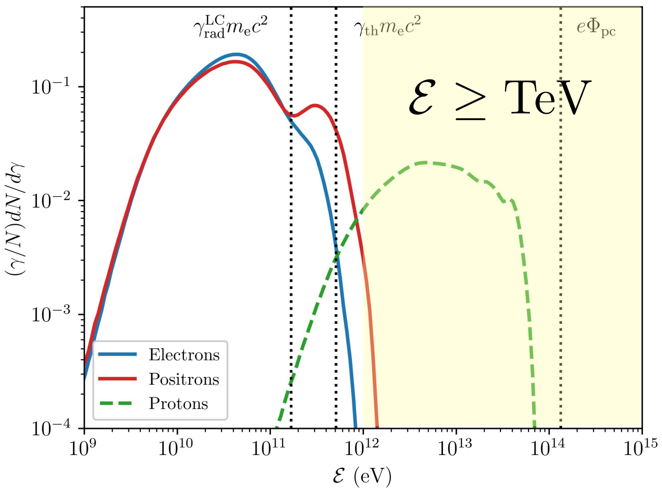
Protons energy



- No radiative cooling
- Acceleration in the current layer

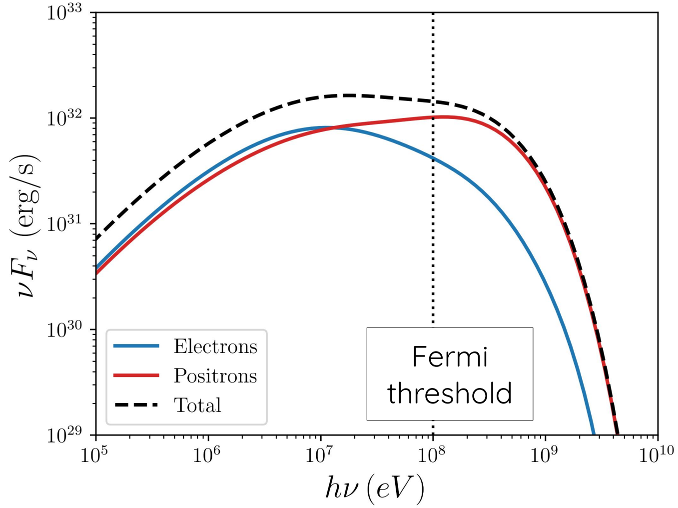
Emission from the current sheet

Particle spectra



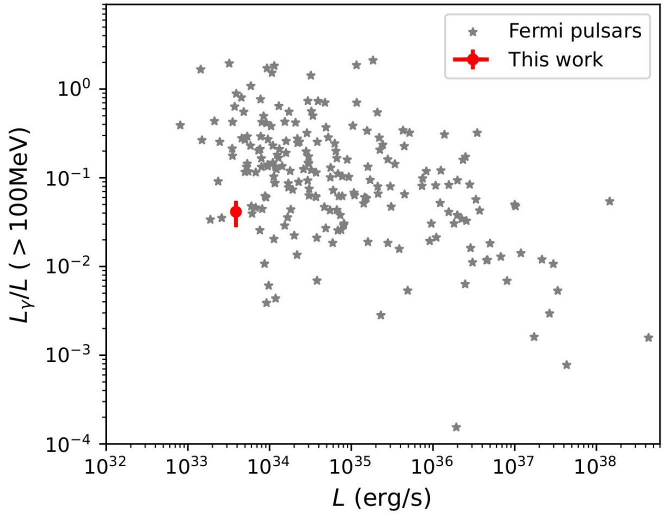
- Leptons:
  - ↪ Similar spectrum
  - ↪ Reach TeV energies
- Protons:
  - ↪ Above TeV (1-30)

Synchrotron spectrum



- Production of γ-rays
- 18% of total spindown
- Observable by *Fermi*
  - ↪ efficiency

Efficiency



- Efficiency ~ 5% / Bulk ~ 20%
- Compatible with observations

Conclusions

Goal: Simulation at scale of a weak Fermi MSP

New numerical method: coupled Force-free-PIC approach

Results:

↳ First global simulation within the detection range of the *Fermi*-LAT satellite
↳ MSP with real scales in a simulation box
↳ TeV accelerators for particles
↳ Emission from the current sheet above Fermi threshold
↳ Direct comparison between simulation and observations
↳ Efficiency in agreement with observations

Upgrades:

↳ realistic pair production
↳ simulation in 3D