Ab initio simulation of neutron star electrospheres

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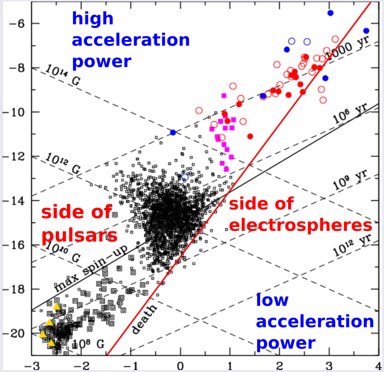
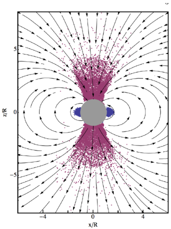
Electrospheres: a few commonplace conjectures

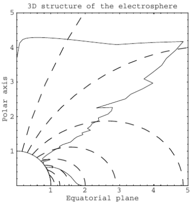


Observed pulsars in the period P and period derivative $\dot{P}$ diagram. Top : stronger magnetic fields. Left : higher spin rates.

- Pulsar = many $e^{-} + e^{+}$ pair creations
- Electrosphere = few (or no) pair creations.
- Electrospheres have tenuous plasma, weak electric currents, they do not radiate strongly
- Electrospheres are not observed
- Electrospheres occur with neutron stars (NS) with low magnetic field and low spin rate ($\sim$ weak energy)
- The pulsar graveyard contains electrospheres, and the huge majority of NS environments .

Previous simulations of electrospheres

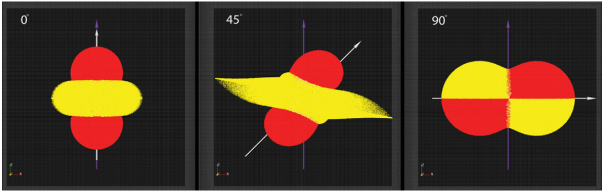




Aligned electrospheres

Left: PIC simulation of ultrafast NS with "indeterminate" magnetic field [Spitkowski+ 2014]

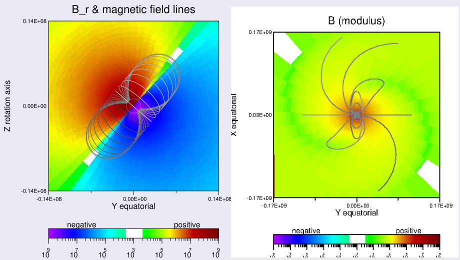
Right: force-free semi analytic, more realistic and detailed study [Petri 2002]



3D PIC simulations, of ultrafast NS with "indeterminate" magnetic field, and force-free inner boundary conditions

[McDonald+Shearer 2009] "initial simulations", with no follow-up. Red: electron density. Yellow: positron density.

Data for a neutron star magnetosphere simulation



At the initial stage of the simulation, the NS surface magnetic field is expanded into vacuum. Left: $i = 45^o$, mid distance. Right: $i = 75^o$ inside and beyond the light cylinder.

- Neutron star (NS) $\leftarrow$ a rotating sphere, with a strong magnetic field, and a surface with a large electrical conductivity.
- The surface magnetic field $B_r(R_*, \theta, \phi)$ is dipolar, with an inclination i to the axis of rotation.
- A surface corotation electric field $\mathbf{E} = -\mathbf{v}_\phi \times \mathbf{B}$ and $\nabla \cdot \mathbf{E} = \rho_{corotation}$.
- The electromagnetic field is non-trivially expanded into vacuum (analytical solution) [Deutch, Petri, Bonazzola+]
- A central electric charge, and the charges surrounding the NS are the other causes of electromagnetic fields.

The *Pulsar ARoMa* code

[Mottez, 2023]

- Stationary solution through iteration process.
- Each iteration include long particle trajectories ($\sim$ 10000 time steps).
- 3D spatial grid in spherical coordinates,
- $\sim$ 10 interlocking spherical shells of variable sizes.
- Inner boundary: $\Delta r \sim 1$ cm. Outer boundary: $\Delta r \sim 100$ m.
- Solve Maxwell's equations with spectral methods (very efficient if soft gradients) [Novak, Petri].
- Particles have a statistical weight depending on their initial energy.
- Trajectories: solve diff. eq. of motion with variable Δt.
- Particle finite inertia $\rightarrow$ parallel motion + guiding center drifts.
- Energy loss by curvature radiation [Vigano 2015].
- No force-free hypothesis (boundaries, motion, EM field).

Pulsar Aroma: like a PIC code, stationary

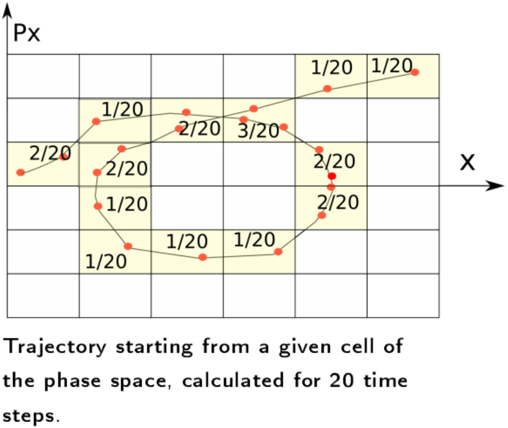
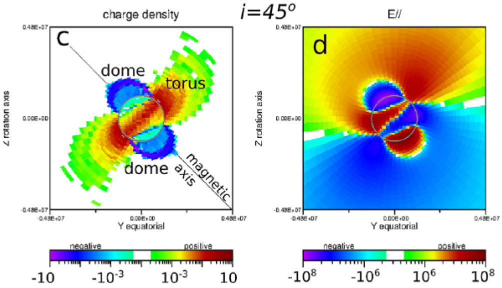


Figure 1: Propagation of a pseudo-particle and deposition of charge and current densities.

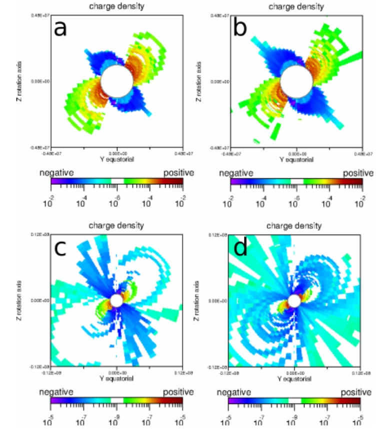
Une electrosphère trop choupi



Inclination $i = 45$ deg, $R = 12$ km, and $Q_b = Q_c$ and $B_1 = 10^9$ G, $P = 10$ ms.

- We can produce oblique electrospheres
- Same structure as in other publications : two electron domes aligned with the magnetic axis + a proton belt.
- Our solutions are not force-free.

We started a parametric study, and cancelled it...



From (a) to (b) and (b) to (c), ΩB_1 is divided by 10. (a,b,c) $Q/Q_c = 0.8$, (d) $Q/Q_c = 0.5$.

- Lower values of ΩB_1 imply larger electron domes.
- Lower value of Q/Q_c imply larger electron domes, or even less confined electrons (as suggested intuitively).

... problem of boundary conditions on the NS surface

1. B_r: given. Arbitrary (here, dipole).
2. Tangential **E**: corotation field
3. E_r: separate vacuum field, and charge density field. [Petri, 2002]
4. Vacuum E_r contains information about NS rotation. Explicit formulas. [Bonazzola 2014, Petri, 2014]
5. Charge density field computed in volume $\nabla\Phi(\rho)$, and $\Phi = 0$ at boundaries.
6. What is the charge density at the NS surface ? We are currently working on it. Above simulations: $\rho(R_*) = \rho_{corotation}$ i.e. continuity.

Emission from primary particles

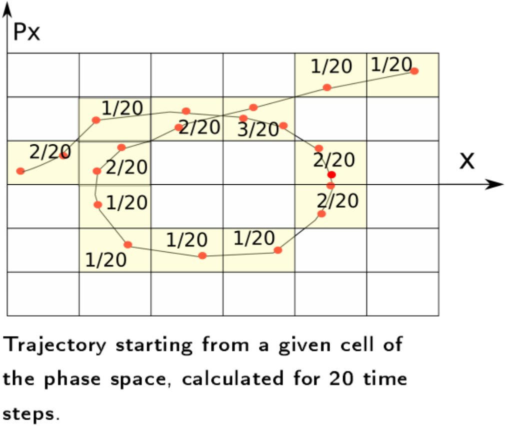


Figure 2: Propagation of a pseudo-particle.

Emission from the primary particles

We solve the radiative transfer equation,

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu \quad , \tag{1}$$

which comes to calculate source terms. In fact, we compute the specific energy E_ν instead of the emissivity j_ν.

- Along a full particle trajectory, curvature radiation is dominant so the energy spectrum of a source term is,

$$E_{\nu\mathrm{so}} = \int_0^{+\infty} \frac{2\sqrt{3}\pi q^2\gamma}{c} \mathrm{F}\left(\frac{\nu}{\nu_c}\right) \frac{\partial N}{\partial \gamma} \mathrm{d}\gamma \quad . \tag{2}$$

Absorption in the magnetosphere

We solve the radiative transfer equation,

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\alpha_\nu I_\nu \quad . \tag{3}$$

The energy absorbed in a cell of volume $\mathrm{d}V$ and absorption coefficient α_ν into which a photon crosses a length $\mathrm{d}s$ is,

$$\mathrm{d}^2 E_{\nu\mathrm{abs}} = \alpha_\nu \underbrace{\left(\frac{I_{\nu\mathrm{so}}}{c}\mathrm{d}V\right)}_{=\mathrm{d}E_{\nu\mathrm{so}}} \mathrm{d}s \quad . \tag{4}$$

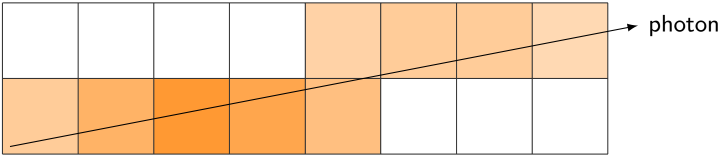


Figure 3: Trajectory of a photon and its ‘peeled off’ radiation.

Origin of α_ν and pair creation

Pair creation, photon-magnetic field channel:

$$\gamma + B \longrightarrow e^+ + e^- \quad .$$

To simplify we take α_ν as an asymptotic reaction rate [Daugherty and Harding, 1983]. In the frame $(\mathscr{R}_0)$ where $\mathbf{k} \cdot \mathbf{B_0} = 0$,

$$\alpha_{\nu 0} = \frac{\alpha B_0'}{\lambda\!\!\!^{-}} \begin{cases} 0.23\mathrm{e}^{-\frac{4}{3\chi_0}} & , \quad \chi_0 \ll 1 \\ 0.30\chi_0^{-\frac{1}{3}} & , \quad \chi_0 \gg 1 \quad , \end{cases} \tag{5}$$

with $\chi_0 = \nu_0' B_0'$, $\nu_0' = \frac{h\nu_0}{2m_e c^2}$ and $B_0' = \frac{B_0}{B_{\mathrm{cr}}}$. The spectrum of the pairs in the frame of the observer $(\mathscr{R})$ is,

$$\frac{\mathrm{d}N}{\mathrm{d}\gamma} = \int \underbrace{\frac{1}{\alpha_\nu}\frac{\mathrm{d}\alpha_\nu}{\mathrm{d}\gamma}}_{\text{probability density}} \underbrace{\frac{E_{\nu\mathrm{abs}}}{h\nu}}_{\text{number of photons}} \mathrm{d}\nu \quad . \tag{6}$$

Population of pair created and synchrotron emission

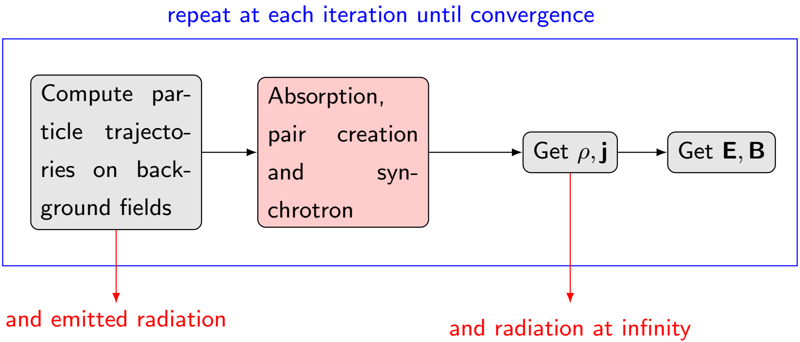
BUT, we treat synchrotron emission right after the creation because the solver does not account for such short scale motion.

$$e^{+} + e^{-} \longrightarrow e^{+} + e^{-} + 2\gamma_{\rm sync} \quad . \tag{7}$$

The synchrotron energy spectrum is [Rybicki, 1979],

$$E_{\nu{\rm sync}} = \int_0^{+\infty} \frac{2\sqrt{3}\pi q^2\gamma}{c} \mathrm{F}\left(\frac{\nu}{\nu_c}\right) \frac{\partial N}{\partial \gamma} \mathrm{d}\gamma \quad . \tag{8}$$

Implementation



Conclusion: what can do Pulsar Aroma ?

- Solve the electrosphere electrodynamics problem with particle distribution functions.
- Solve the radiative transfer in the magnetosphere with the help of distribution functions : emission from particles, absorption by pair creations.
- We can parametrize the NS with realistic parameters, at moderate computing costs.
- Injection of particles from the NS: work in progress.

Thank you for your attention !

References I

Daugherty, J. K. and Harding, A. K. (1983).
Pair production in superstrong magnetic fields.
The Astrophysical Journal, 273:761–773.

Rybicki, B. G. (1979).
Radiative processes in astrophysics.
A Wiley-Interscience publication. J. Wiley and sons, New York Chichester Brisbane [etc.