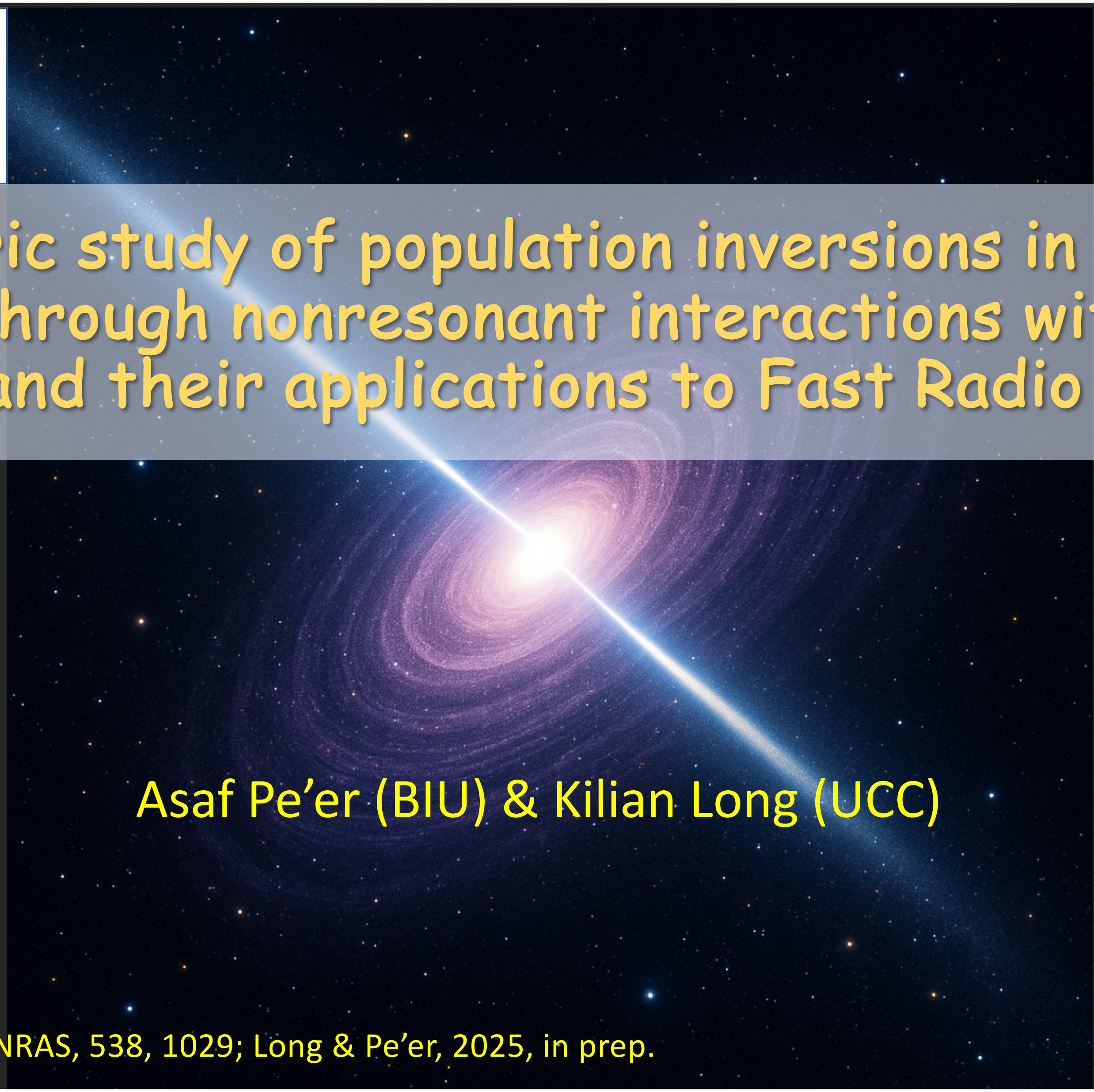

A parametric study of population inversions in relativistic plasmas through nonresonant interactions with Alfvén waves and their applications to Fast Radio Bursts

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Long & Pe'er, 2025, MNRAS, 538, 1029; Long & Pe'er, 2025, in prep.

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Explaining FRBs: the synchrotron maser

- FRB brightness temperature: ~10^{36} k

- Conditions for sync. maser:
  - electron population inversion
  - strong magnetic field

  interaction between pop. inverted e + EM waves → stimulated emission (maser)

Lyubarsky, 2014
Long & Pe'er, 2018
Metzger et. al., 2019

- <u>Population inversion:</u> $\frac{\partial F}{\partial v_\perp} > 0$ <u>- does not require a shock wave !</u>

Non-resonant interaction between Alfvenic waves and relativistic (hot) particles: complete formula

(Stix, 92; Yoon+09)

← To NS

T, n, $B_0 \rightarrow \sigma = \Omega^2/\omega_p^2$

$\Omega = eB_0/mc$

$F_0(p_\parallel, p_\perp, \theta_0)$ = Maxwell-Juttner

$$\frac{\partial F}{\partial t} = \frac{e^2}{4}\sum_{l=\pm1}\int d\mathbf{k}\frac{1}{p_\perp}\left[\left(1-\frac{k_\parallel p_\parallel}{\gamma m\omega}\right)\frac{\partial}{\partial p_\perp}+\frac{k_\parallel p_\perp}{\gamma m\omega}\frac{\partial}{\partial p_\parallel}\right]$$
$$\times\left\{p_\perp\left[\pi\delta\left(\omega-l\omega_c-\frac{k_\parallel p_\parallel}{\gamma m}\right)|E_k|^2\right.\right.$$ resonance
$$\left.-\frac{\partial}{2\partial\omega}\left(PV\left(\frac{1}{\omega-l\omega_c-\frac{k_\parallel p_\parallel}{\gamma m}}\right)\right)\frac{\partial|E_k|^2}{\partial t}\right]$$ non res.
$$\left.\times\left[\left(1-\frac{k_\parallel p_\parallel}{\gamma m\omega}\right)\frac{\partial}{\partial p_\perp}+\frac{k_\parallel p_\perp}{\gamma m\omega}\frac{\partial}{\partial p_\parallel}\right]F\right\}. \qquad (1)$$

<u>Non-resonance condition:</u>

Origin : starquakes

$K \sim 2\pi/\xi R_*$, ξ – fraction of NS radius

Waves in NS crust: $\xi \sim 10^{-2} - 10\ R_{*,6}^{-1}$

$K \| B_0$ – Alfven wave vector, ω its freq.

Strong B_0 field: $\omega_c = \Omega/\gamma \gg \{\omega, \frac{k_\parallel p_\parallel}{\gamma m}\}$

→ No resonance

Non-resonant interaction between Alfvenic waves and relativistic (hot) particles: strong B field

Any T (relativistic/ non. rel),
l=+-1 harmonics

$$
\begin{aligned}
\frac{\partial F}{\partial t} = \frac{7.7\times 10^{-4}}{\omega_{c,9}^2}\Bigg\{ &\left(\frac{I_1\left(2+\frac{q_\perp^2}{\gamma^2}\right)}{\gamma} - 2\frac{I_2 q_\parallel}{\gamma^2}\right)\frac{\partial F}{\partial q_\parallel} \\
&+\left[\frac{1}{q_\perp}\left(I_3\left(1+2\frac{q_\perp^2}{\gamma^2}\right)+\frac{I_2 q_\parallel^2}{\gamma^2}-\frac{I_1 q_\parallel\left(2+\frac{q_\perp^2}{\gamma^2}\right)}{\gamma}\right)\right. \\
&\left.-\frac{I_2 q_\perp}{\gamma^2}\right]\frac{\partial F}{\partial q_\perp} \\
&+\frac{I_2 q_\perp^2}{\gamma^2}\frac{\partial^2 F}{\partial q_\parallel^2} \\
&+\left(I_3+\frac{I_2 q_\parallel^2}{\gamma^2}-2\frac{I_1 q_\parallel}{\gamma}\right)\frac{\partial^2 F}{\partial q_\perp^2} \\
&+\left(2\frac{I_1 q_\perp}{\gamma}-2\frac{I_2 q_\perp q_\parallel}{\gamma^2}\right)\frac{\partial^2 F}{\partial q_\parallel \partial q_\perp}\Bigg\},
\end{aligned}
\tag{3}
$$

Parallel advection

Perpendicular advection

Parallel diffusion

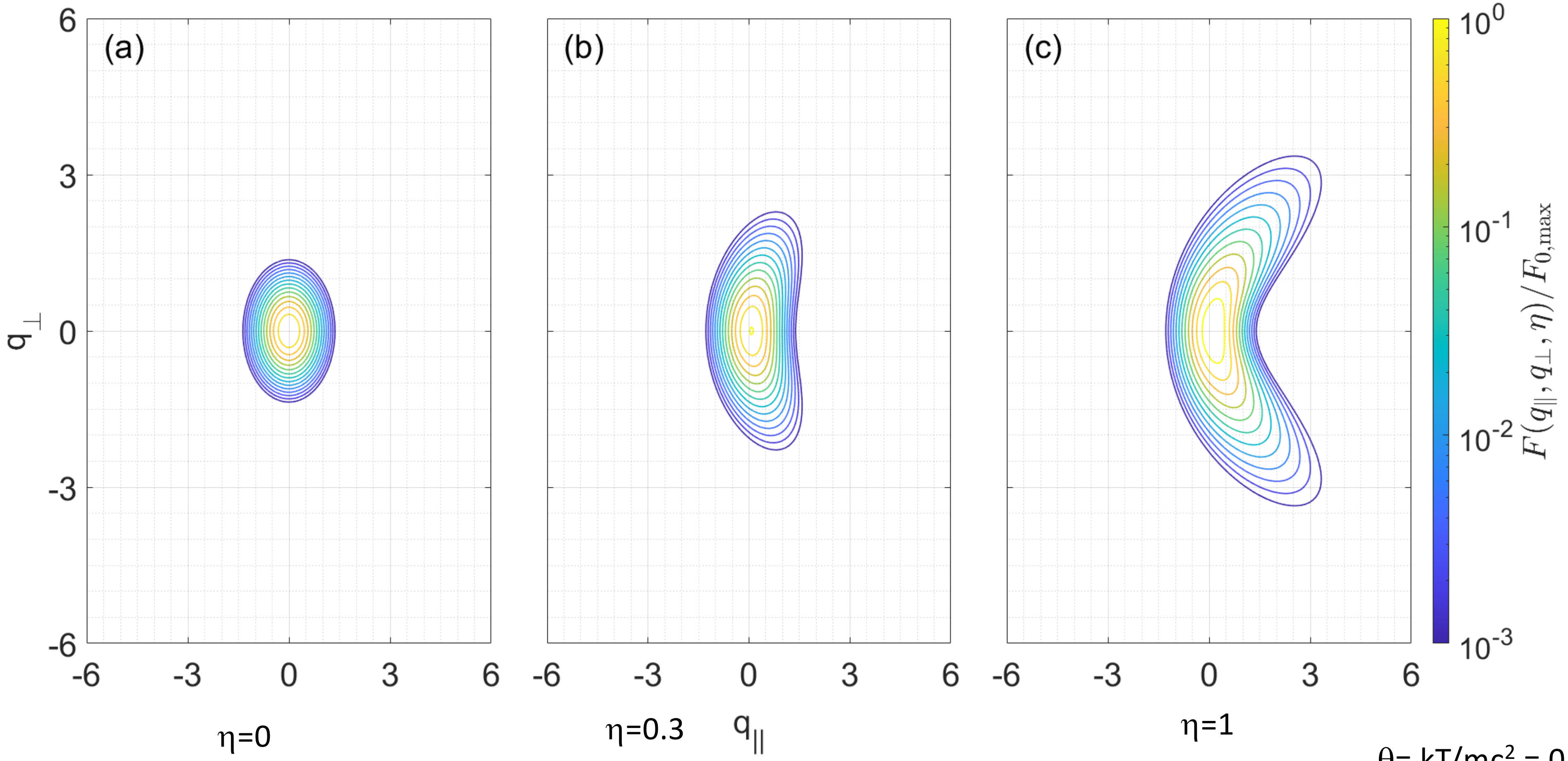
Perpendicular diffusion

Mixed

where the factor $\frac{7.7\times10^{-4}}{\omega_{c,9}^2} = \frac{e^2}{4c^2m^2\omega_c^2}$, $q = p/mc = \gamma\frac{v}{c} = \gamma\beta$, and $I_1 = \int d\mathbf{k}\frac{\partial|E_k|^2}{\partial t}\frac{ck_\parallel}{\omega}$, $I_2 = \int d\mathbf{k}\frac{\partial|E_k|^2}{\partial t}\frac{c^2k_\parallel^2}{\omega^2}$ and $I_3 = \int d\mathbf{k}\frac{\partial|E_k|^2}{\partial t}$. Equation (3) is correct in the limit of strong

Hierarchy: $I_2 > I_1 > I_3$ ($v_A < c$)
[$I_2/I_1 = c/v_A \rightarrow 1$]

Formation of a population inversion



θ= kT/mc^2 = 0.1

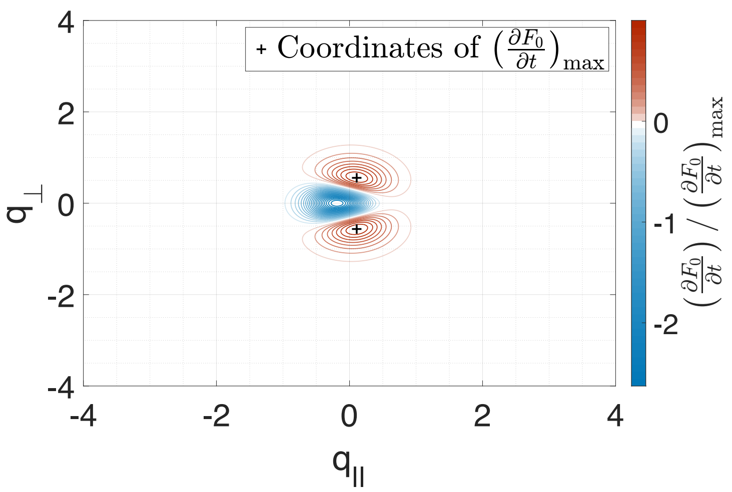
$\eta = \frac{\int dk\, B_k^2}{B_0^2}$ Alfven turbulence level

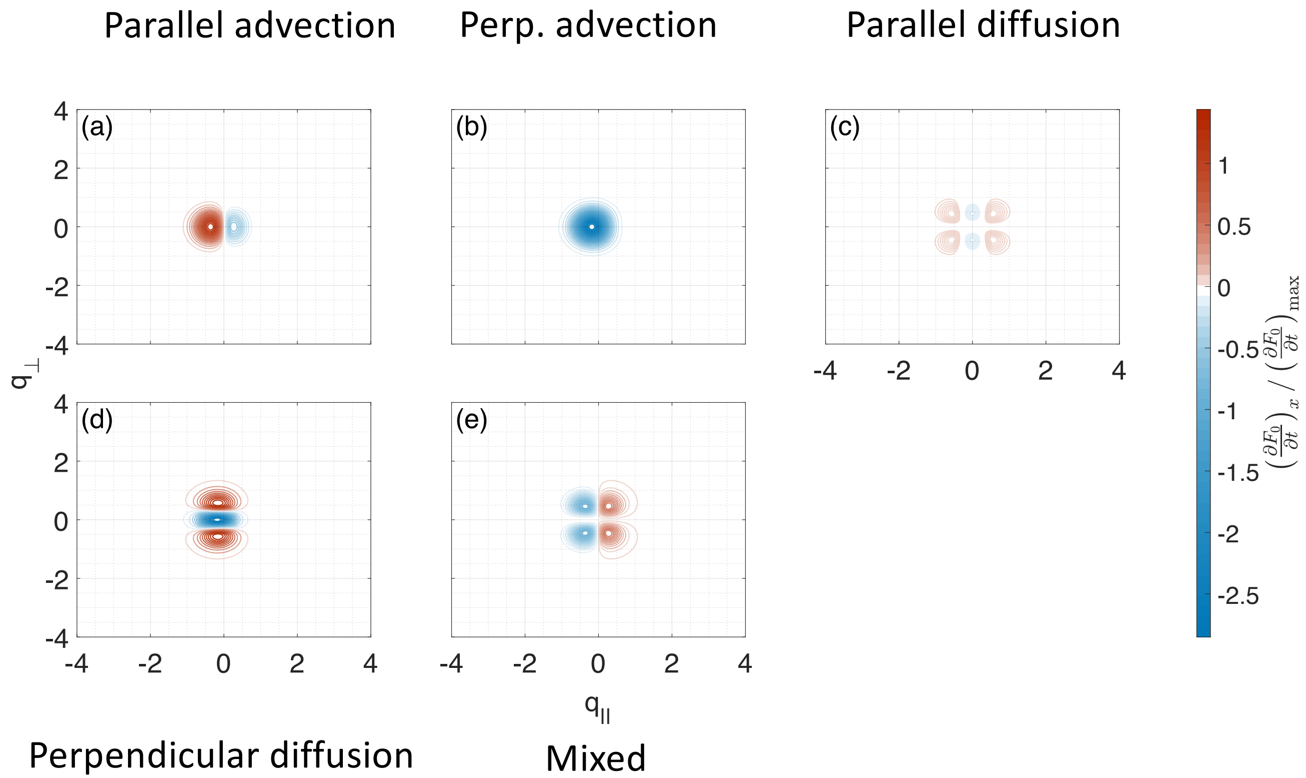
$E_k(t) \sim k^{-\phi} e^{\Gamma_k t}$ Set (only) the time scale

I_2/I_1 = 1 → σ>~10

Formation of a population inversion

Dist. change, $\partial F_0/\partial t$





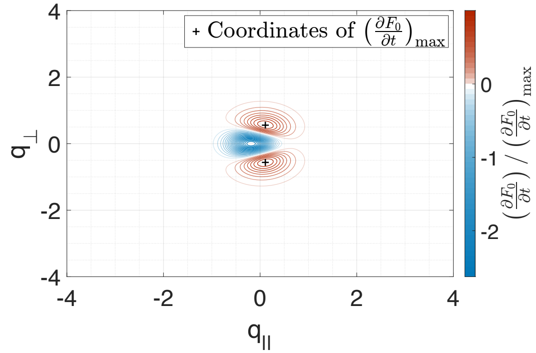
$\theta = kT/mc^2 = 0.1$

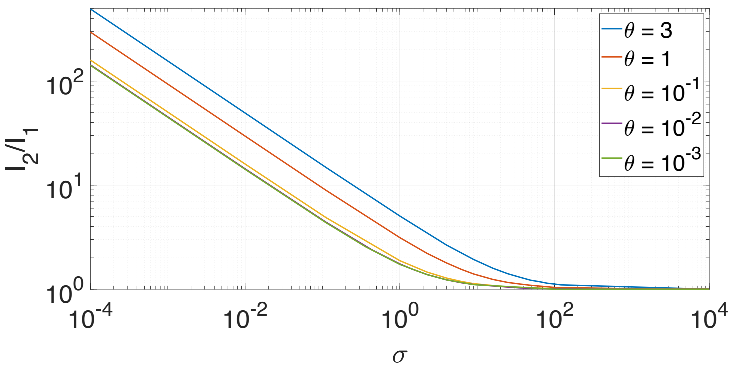
$I_2/I_1 = 1 \rightarrow \sigma >\sim 10$

Level of population inversion (crescent shape) is determined by the mixed term

Population inversion: magnetization

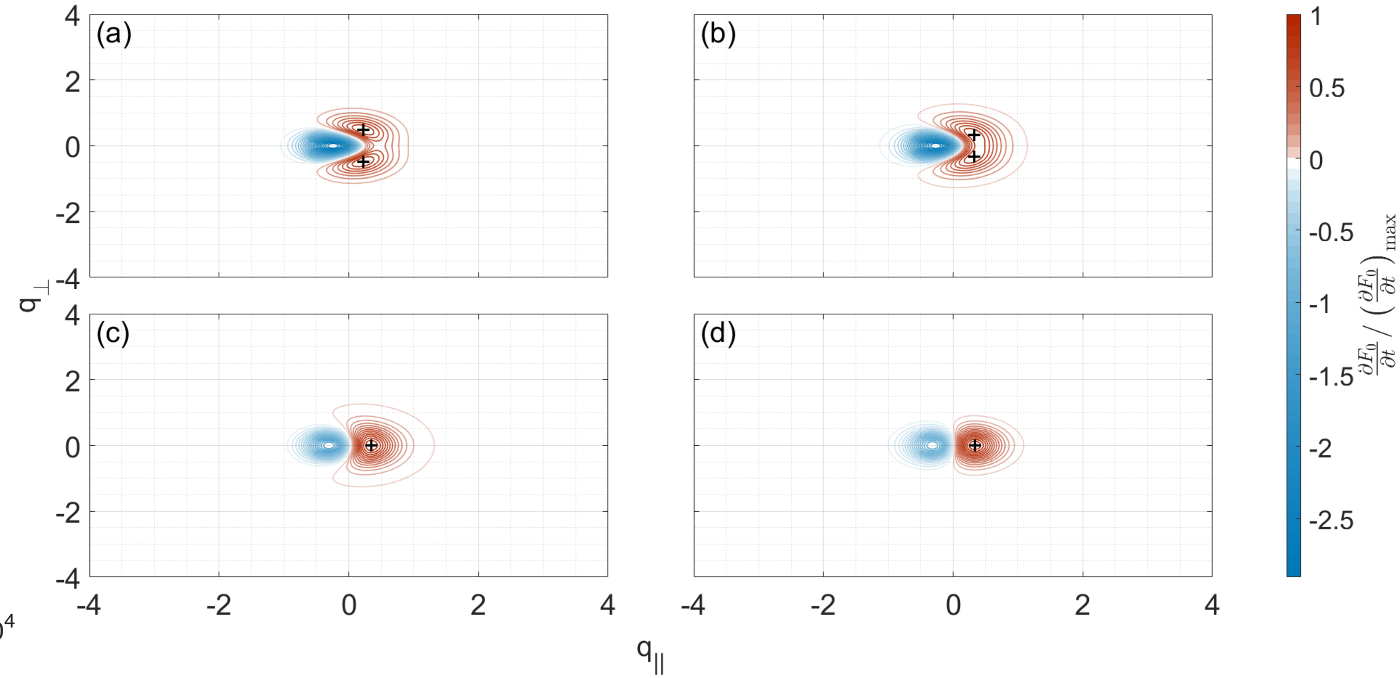
Dist. change, $\partial F_0/\partial t$





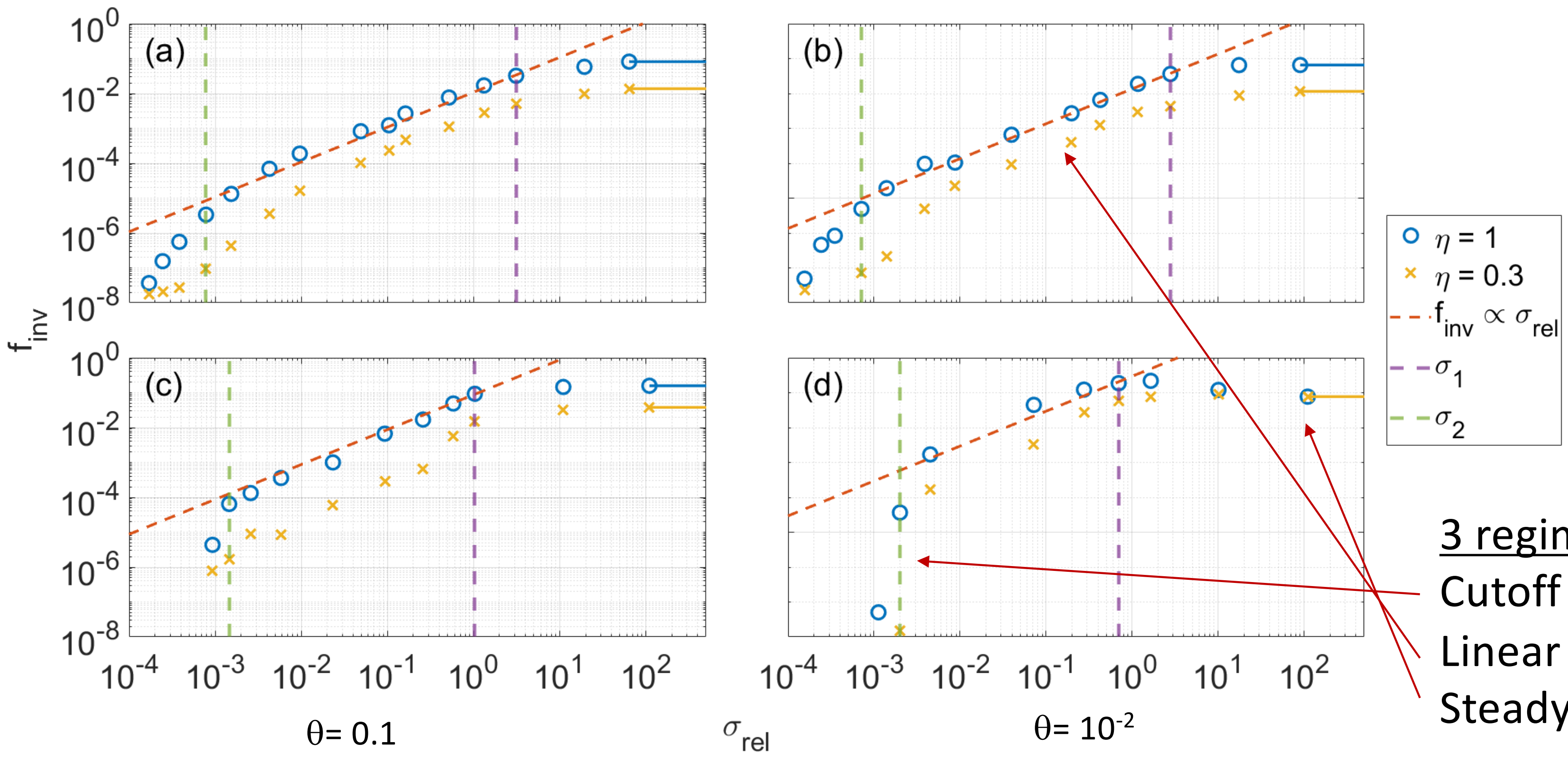
$l_2/l_1 = c/v_A$

$\theta = kT/mc^2 = 0.1$



Lower magnetization – higher ratio l_2/l_1 – par. advec. dominates – less inversion

Results: fraction of energy available for masing



$$f_{inv} = \frac{E_{tot} - E_{bi}}{E_{tot}}$$

$$E_{tot} = \int dp \, (\gamma(p) - 1) F(p)$$

<u>3 regimes:</u>

Cutoff (particles spread)

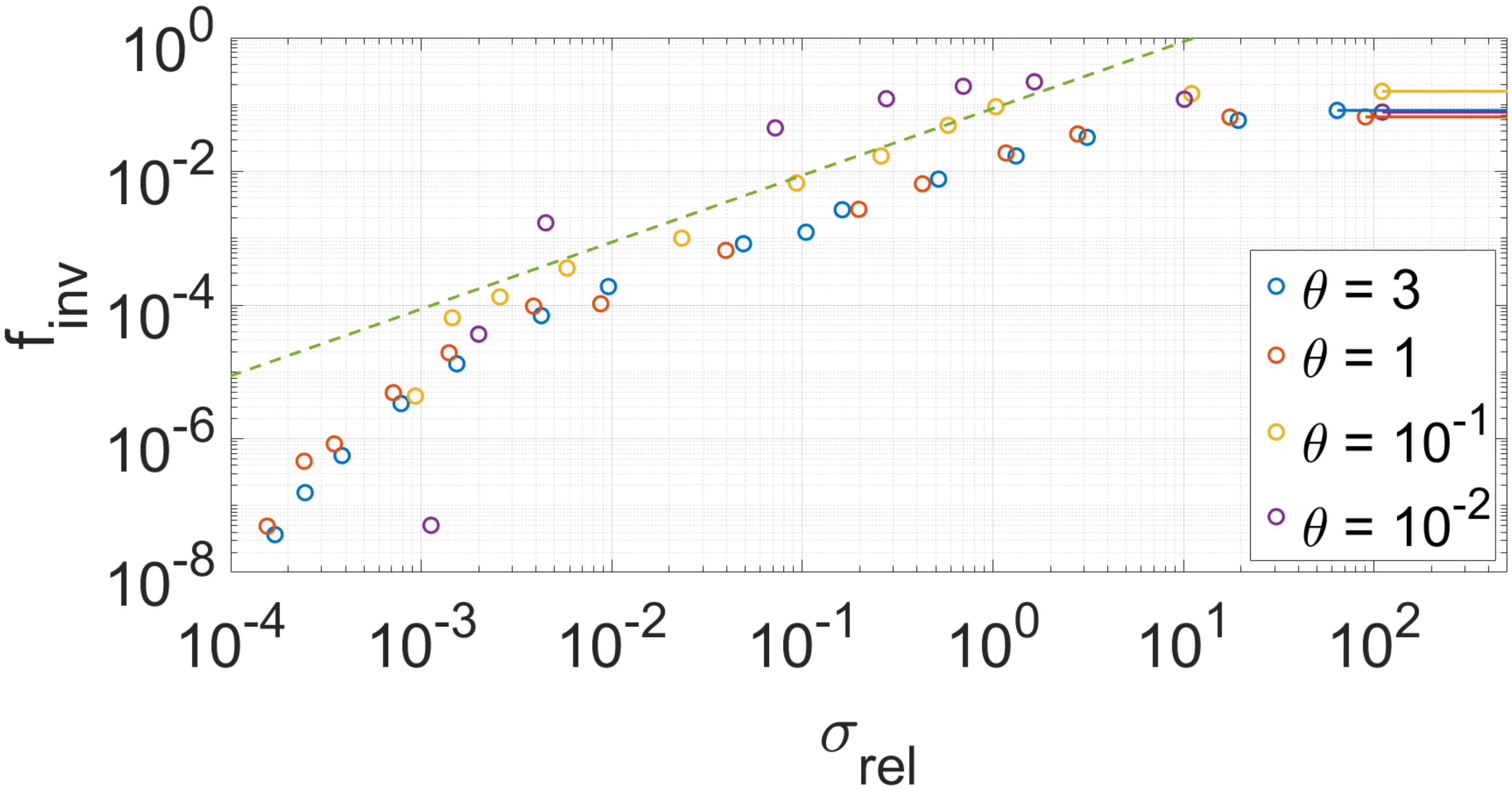
Linear (mix term $\propto \sigma_{rel}$)

Steady ($I_1:I_2:I_3$ = fixed)

High magnetization ($\sigma>1$) → f_{inv} >~10^{-2}

$\sigma_{rel} = \sigma / \gamma_{avg}$

Results: fraction of energy available for masing



$$f_{inv} = \frac{E_{tot} - E_{bi}}{E_{tot}}$$

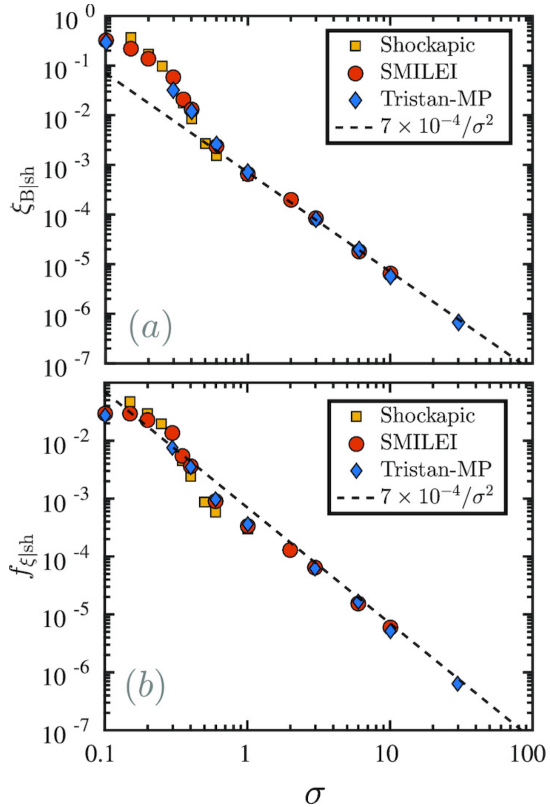
$$E_{tot} = \int dp\, (\gamma(p) - 1)F(p)$$

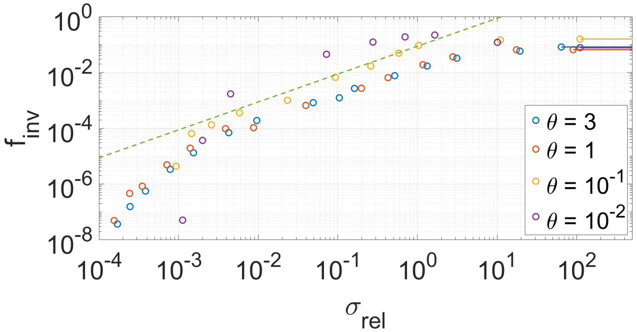
f_{inv} reach few %; slightly decreases with temperature

Comparative models

Non-resonant interactions	Relativistic shocks (PIC)
	Particle gyrating around solitons formed near shock front (enhanced magnetic field)
High magnetization: f_{inv} >~10^{-2}	f_{inv} ~$10^{-3}\,\sigma^{-1}$
Intermediate: f_{inv} ~$2*10^{-2}$ (θ>=10^{-2})	f_{inv} ~10^{-1}

Plotnikov & Sironi, 21

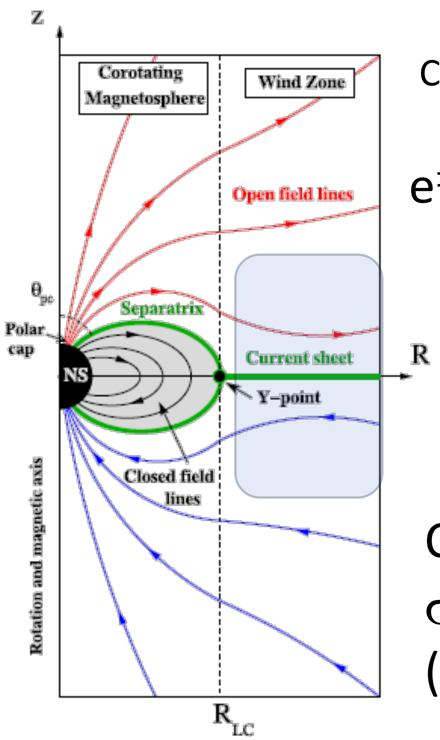




Long & Pe'er, 25

Physical scenario: pair wind outside the light cylinder

light cylinder R_{LC} = $cP/2\pi = 5 * 10^9$ P cm



Cerutti & Beloborodov, 2017

$e^{\pm}$ flux: $\dot{N}$~$3*10^{39}\mu_{33}^{2/3}$ P^{-1} s^{-1}

Beloborodov, 2020

Constraint on density:

$\sigma_{rel}>1 \rightarrow n_{max}$ ~10^{10} $\nu_9^2\gamma_{av}$ Γ_B^{-2} cm^{-3}

(similar for $\sigma<1$, with $\gamma_{av} \rightarrow \sigma^{1/2}$)

Maser's peak frequency: (Lyubarsky, 21)

ω_m~$\Omega\gamma_{av}^{-1}$

For plasma at L.F. Γ_B:

$\rightarrow B_0$ ~360 $\nu_9\gamma_{av}\Gamma_B^{-1}$ G

--vs--

Magnetization in magnetar wind at R_{LC}:

σ_{wind} ~280 $B_{*,15}^{8/9}$ $R_{*,6}^{24/9}$ $M_3^{-2/3}$ P^{-2} >> σ_1

- Conditions for high σ satisfied

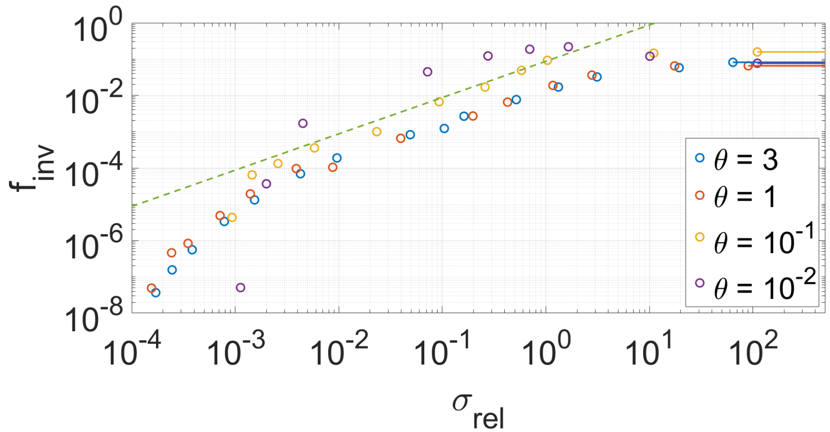
Prediction: For B~1/R at R>R_{LC}, R_{FRB} ~ 10^{11} P^{-2} cm >=R_{LC} , smaller than for shocks

-consistent with the restriction due to damping (see Sobacchi's talk) for Γ_B >>1

Summary

- Non-resonant interaction between Alfven waves and hot (relativistic) plasma ($\theta>10^{-2}$) produce population inversion for $\sigma>10^{-4}$.
- Total energy fraction f_{inv}~0.1 for high magnetization, $\sigma>10$
- Time scale to reach inversion: $t\sim\Gamma^{-1}$ s – comparable with magnetar period

- Model for synchrotron maser emission in FRBs: Cloud at $R^{FRB} \sim 10^{11}$ cm>~ R_{LC}, $n <\sim n_{max} \sim 10^{10} \nu_9^2 \gamma_{av} \Gamma_B^{-2}$ cm^{-3}



✓ Higher frequencies: **suppressed**, as require masing inside the light cylinder, where η is small (B_0 large inside magnetosphere) – no (small) population inversion

✓ Lower frequencies: **suppressed**, as Alfven wave freq. ~ Ω → no non-resonant interaction

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