Particle-in-cell electromagnetic solver for QED polarization in super-strong magnetic fields approach

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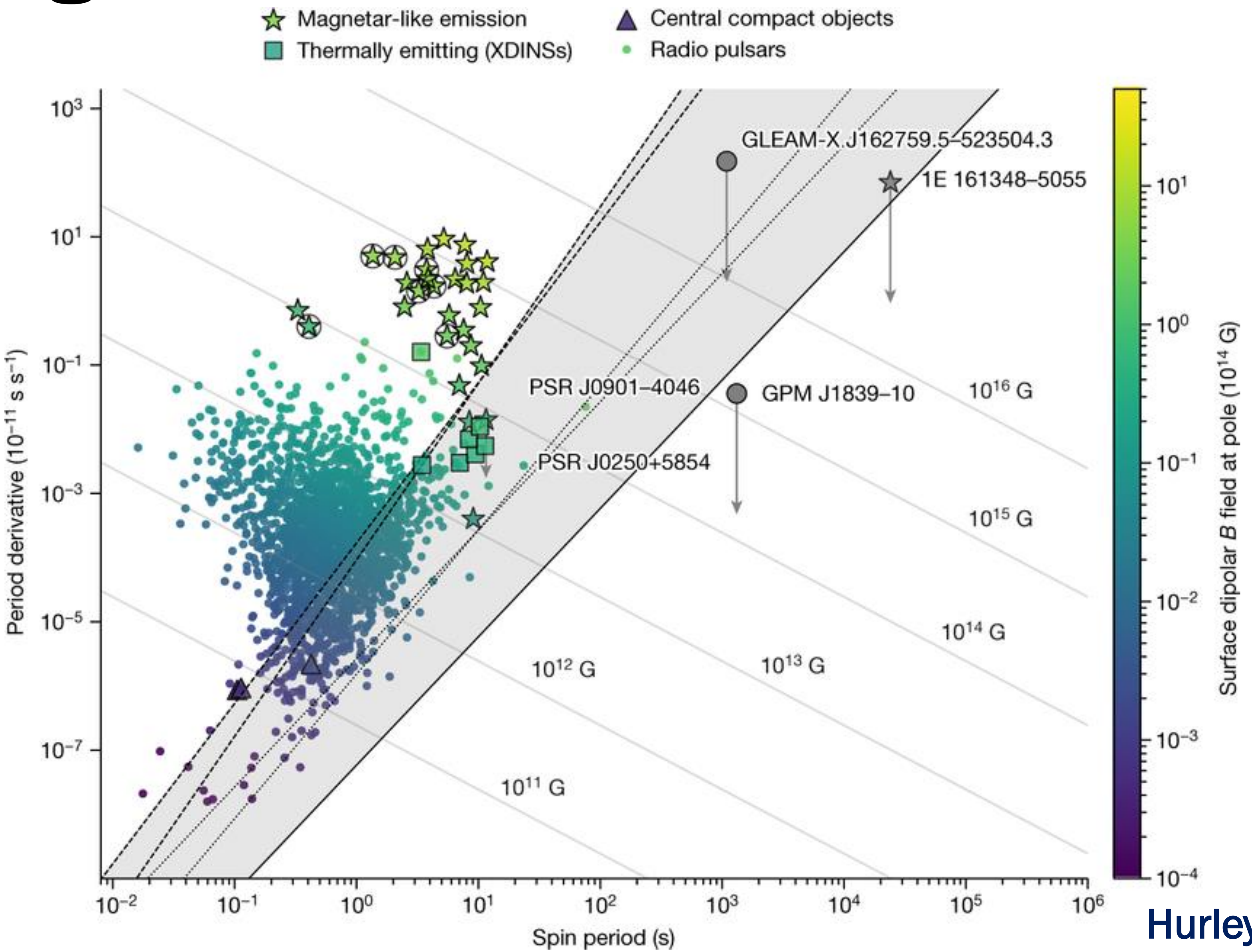
Outline

- Introduction
- Nonlinear Maxwell's equations
- Implementation in PIC
- Validation and results

Introduction

Magnetars have surface magnetic field strengths exceed 10^{14} G



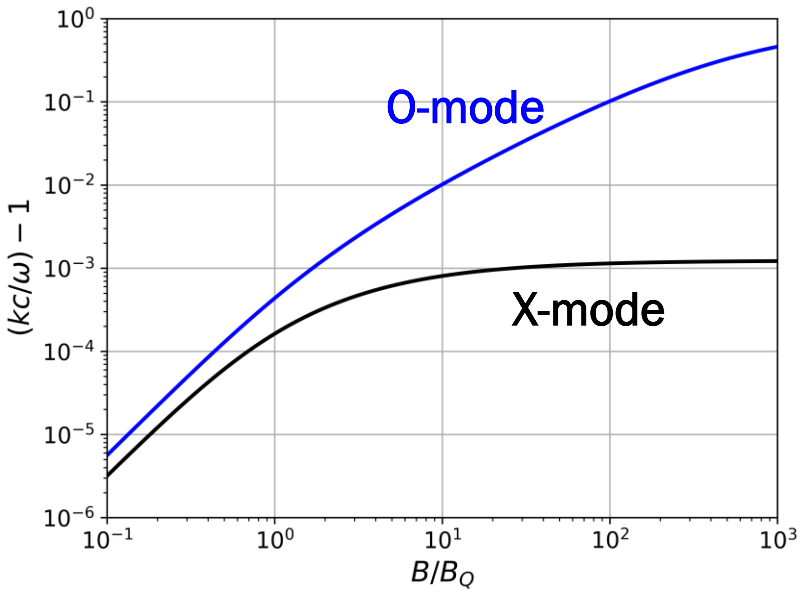
Hurley-Walker et al (2023)

QED effects become relevant above the quantum fields limit:

$$E_Q = {m_e}^2 c^3/\hbar e$$

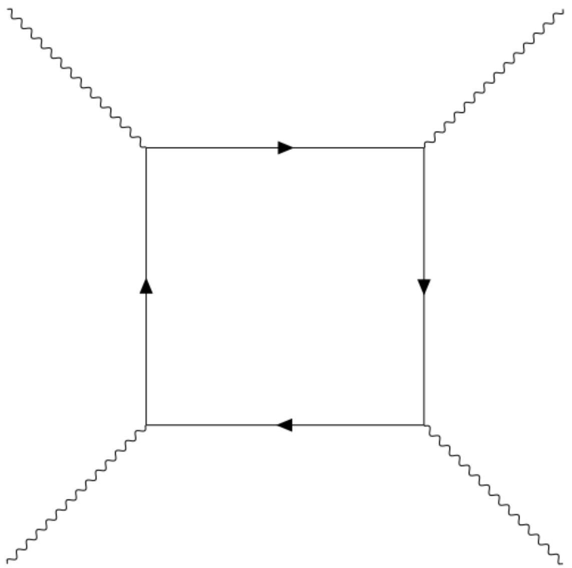
$$B_Q = {m_e}^2 c^3/\hbar e \sim 4.4 \times 10^{13}\ \mathrm{G}$$

Quantum birefringence



Electric field either **parallel (O-mode)** or **perpendicular (X-mode)** to the $\boldsymbol{k}$-$\boldsymbol{B}$ plane

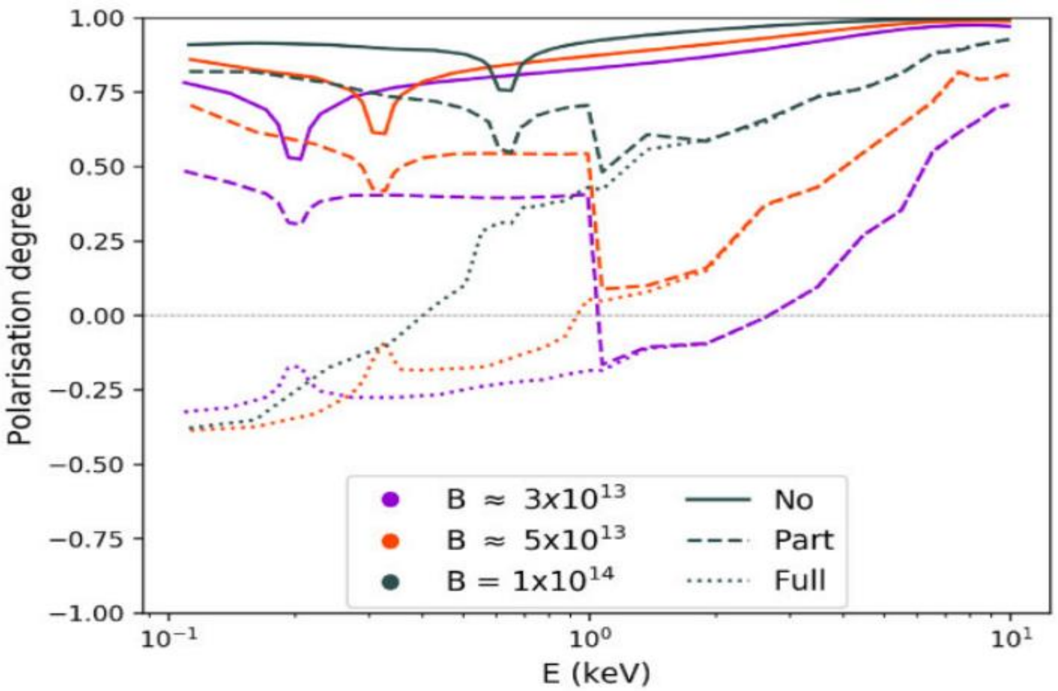
Photon-photon scattering



QED effect in pulsars and magnetars

- QED corrections result in the deflection of the propagation and the change of the polarization mode of the waves propagating in the pulsar magnetosphere. (D. H. Kim, C. M Kim and S. P. Kim (2024))

- Mode conversion at the vacuum resonance could be at play for the magnetars observed by IXPE. (Lai (2023), Kelly et al. (2024), Taverna and Turolla (2024))



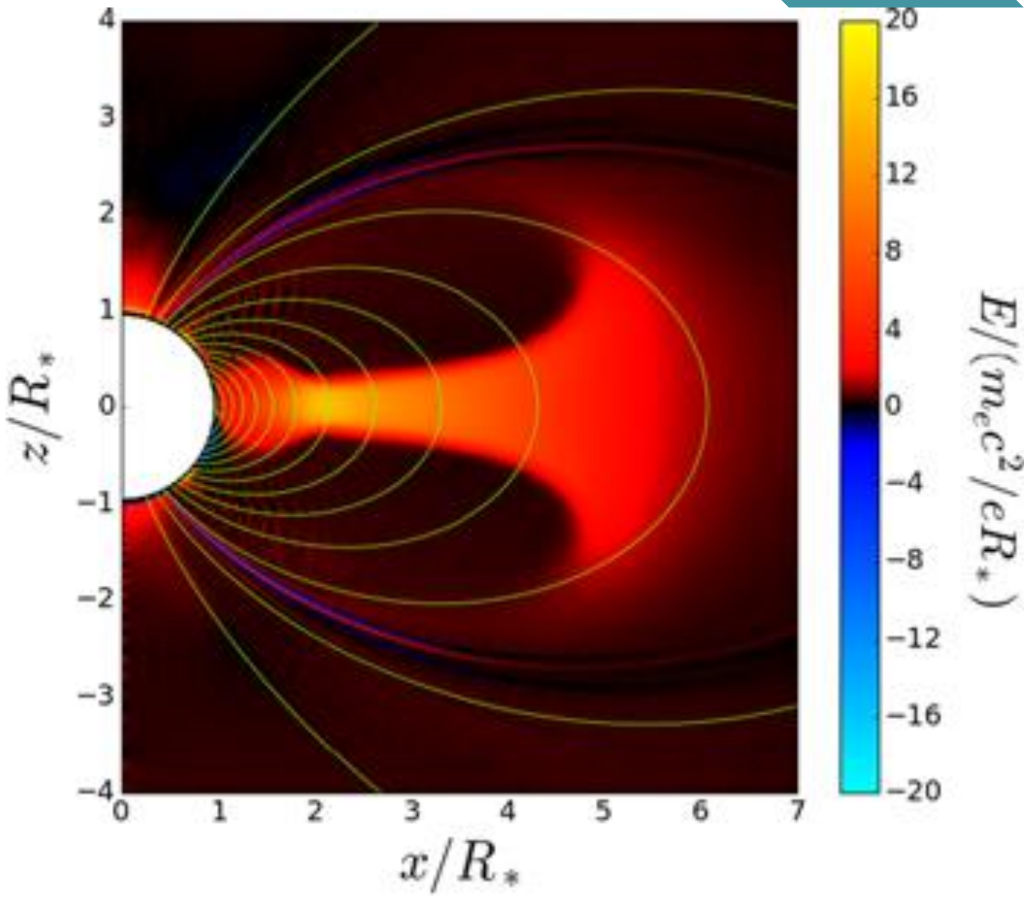
Kelly et al. (2024)

PIC studies of the NS Magnetosphere

- Pulsars Polar-Cap and Magnetosphere using PIC simulations (Cruz+2021b, Benáček+ 2024b, Chernoglazov 2024).

- Large-scale, realistic simulation of a magnetar's twisted magnetosphere (Chen & Beloborodov (2017)).

- PIC simulations use Linear Maxwell's equations: One can extend it to **non-linear Maxwell's equations** for the super-strong magnetic fields.



Chen & Beloborodov (2017)

Nonlinear Maxwell's equations

QED correction in the one-loop approximation

- The Heisenberg-Euler Lagrangian density encapsulates all orders of the one-loop photon-photon interaction:

$$\mathcal{L}_{\mathrm{HE}} = \frac{m_{\mathrm{e}}c^2}{8\pi^2}\left(\frac{m_{\mathrm{e}}c}{\hbar}\right)^3 \int_0^\infty \frac{e^{-\eta}}{\eta^3}\left[-(\eta a \cot \eta a)(\eta b \coth \eta b) + 1 - \frac{\eta^2}{3}(a^2 - b^2)\right] \mathrm{d}\eta$$

- The parameters are

W. Heisenberg and H. Euler (1936)

$$a = \frac{E}{E_{\mathrm{Q}}}, \qquad b = \frac{B}{B_{\mathrm{Q}}},$$

- The general analytical Euler–Lagrange equations are **not known.** Analytical solutions are only possible for the case of an arbitrary strong magnetic field and a very small electric field and vice versa. (J. Lundin (2010), M. V. Medvedev (2023))

Nonlinear Maxwell's equations
strong magnetic field case

- The first pair of Maxwell's equations, $\partial_\nu \hat{F}^{\mu\nu} = 0$, is unaffected by the QED corrections:

$$\nabla \cdot \mathbf{B} = 0,$$

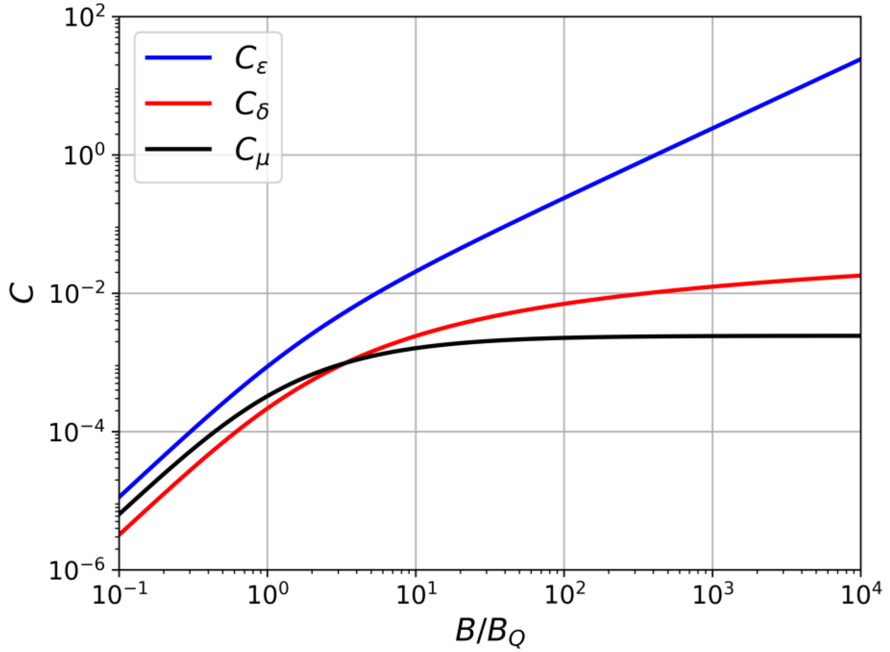
$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0.$$

Nonlinear Maxwell's equations
strong magnetic field case

- Modified Gauss' law:

$$(-1 + C_\delta)\,\nabla \cdot \mathbf{E} + \frac{C_\mu}{B^2}\,\mathbf{E} \cdot \nabla\left(\frac{\mathbf{B}^2 - \mathbf{E}^2}{2}\right) + \frac{C_\epsilon}{B^2}\,\mathbf{B} \cdot \nabla(-\mathbf{B} \cdot \mathbf{E}) = -4\pi\rho$$

- The coupling scalars are magnetic field dependence

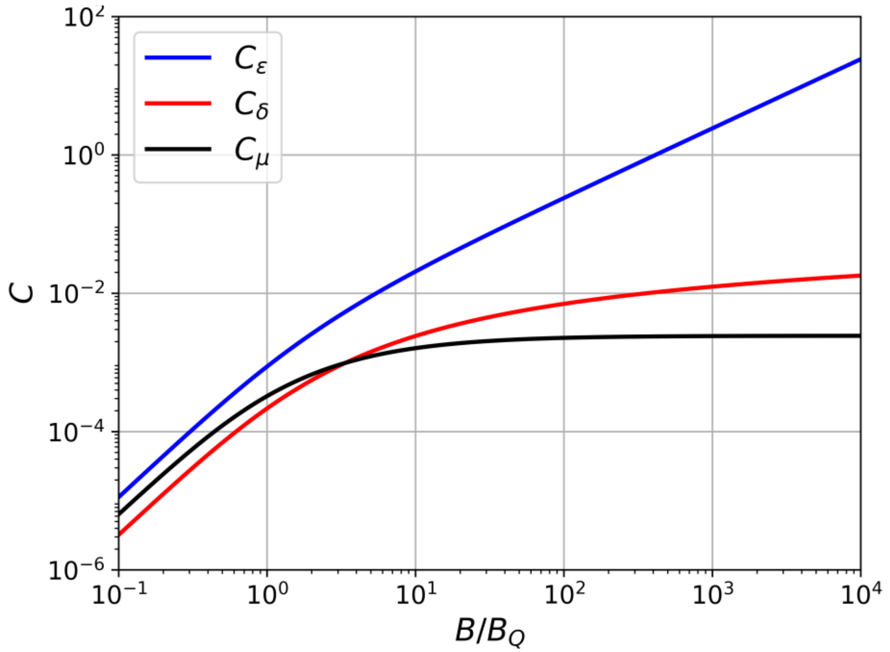


Nonlinear Maxwell's equations
strong magnetic field case

- Modified Ampere's law:

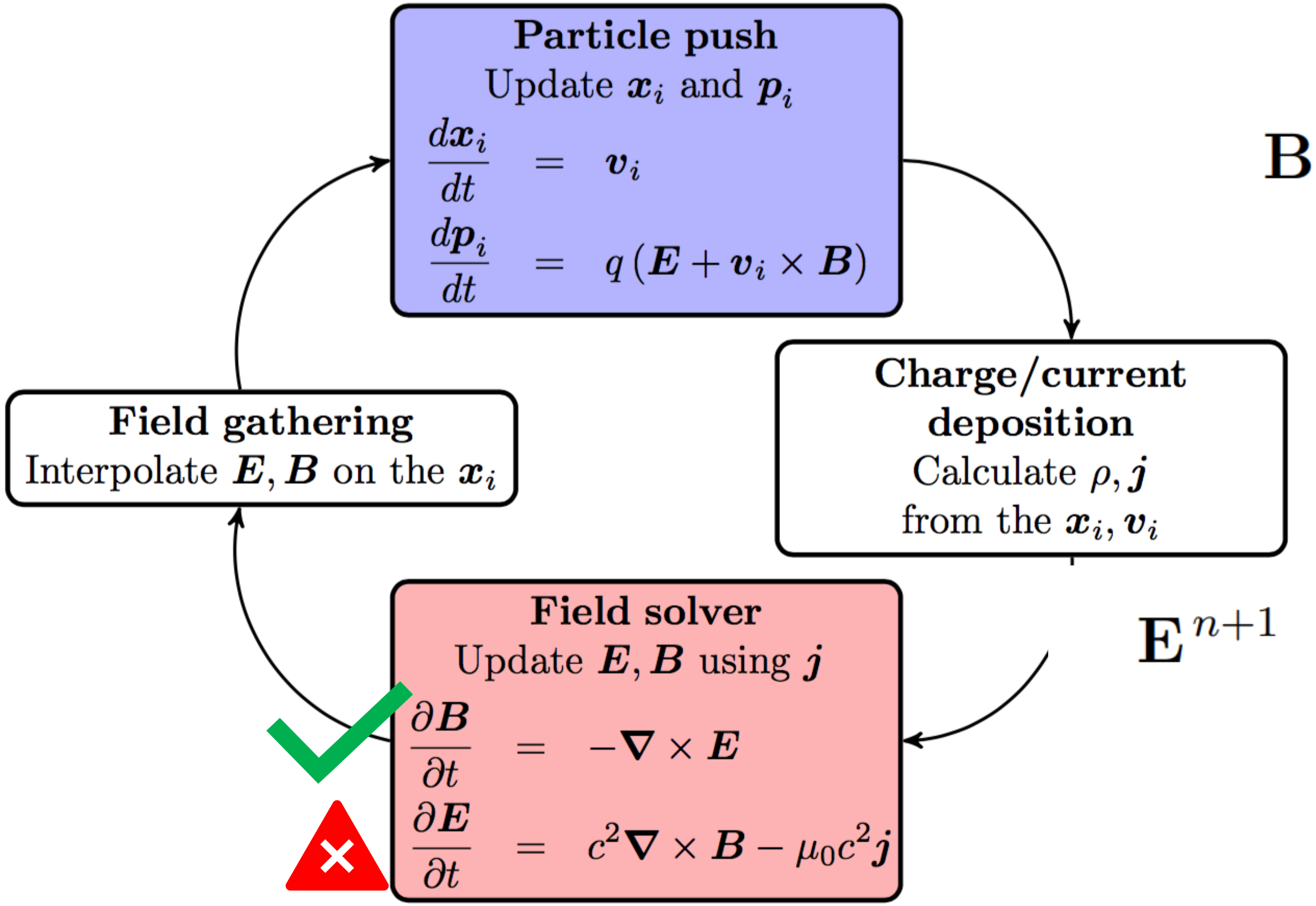
$$(-1+C_\delta)\left[\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}-\nabla\times\mathbf{B}\right]+\frac{C_\mu}{B^2}\left[\mathbf{E}\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\mathbf{B}^2-\mathbf{E}^2}{2}\right)+\mathbf{B}\times\nabla\left(\frac{\mathbf{B}^2-\mathbf{E}^2}{2}\right)\right]$$
$$+\frac{C_\epsilon}{B^2}\left[\mathbf{B}\frac{1}{c}\frac{\partial}{\partial t}(-\mathbf{B}\cdot\mathbf{E})-\mathbf{E}\times\nabla(-\mathbf{B}\cdot\mathbf{E})\right]=\frac{4\pi}{c}\mathbf{j}.$$

- The coupling scalars are Magnetic field dependence



Implementation in PIC

PIC loop modification



$$\mathbf{B}^{n+1/2} = \mathbf{B}^{n-1/2} - c\Delta t(\nabla \times \mathbf{E})^{n}$$

Faraday's law holds

$$\mathbf{E}^{n+1} = \mathbf{E}^{n} + c\Delta t(\nabla \times \mathbf{B})^{n+1/2} - 4\pi \mathbf{j}^{n+1/2}$$

Ampere’s law doesn't hold

Nonlinear Ampere's law
strong magnetic field case

$$\gamma_{\mathscr{F}}\left[\frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}-\nabla\times\mathbf{B}\right]+\gamma_{\mathscr{F}\mathscr{F}}\left[\mathbf{E}\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\mathbf{B}^2-\mathbf{E}^2}{2}\right)+\mathbf{B}\times\nabla\left(\frac{\mathbf{B}^2-\mathbf{E}^2}{2}\right)\right]$$
$$+\gamma_{\mathscr{G}\mathscr{G}}\left[\mathbf{B}\frac{1}{c}\frac{\partial}{\partial t}(-\mathbf{B}\cdot\mathbf{E})-\mathbf{E}\times\nabla(-\mathbf{B}\cdot\mathbf{E})\right]=\frac{1}{c}\mathbf{j}.$$

$$\gamma_{\mathscr{F}}=-(1-C_\delta)/(4\pi),$$
$$\gamma_{\mathscr{F}\mathscr{F}}=C_\mu/(4\pi B^2),$$
$$\gamma_{\mathscr{G}\mathscr{G}}=C_\varepsilon/(4\pi B^2),$$

J. Lundin (2010)
M. V. Medvedev (2023)

QED polarization field solver

$$\frac{\partial \mathbf{E}}{\partial t} = A^{-1}\left(\frac{1}{c}\mathbf{j} - \mathbf{Q}\right)$$

$$A_{ij} = \frac{1}{c}\left[\gamma_{\mathcal{F}}\delta_{ij} - \gamma_{\mathcal{FF}}E_iE_j - \gamma_{\mathcal{GG}}B_iB_j\right]$$

$$\gamma_{\mathcal{F}} = \frac{-1 + C_\delta(b)}{4\pi} \qquad \gamma_{\mathcal{FF}} = \frac{C_\mu(b)}{4\pi B^2} \qquad \gamma_{\mathcal{GG}} = \frac{C_\epsilon(b)}{4\pi B^2}$$

$$\begin{aligned}
Q_{\mathrm{x}} = &\gamma_{\mathcal{F}}\left[-\left(\frac{\partial B_{\mathrm{z}}}{\partial y} - \frac{\partial B_{\mathrm{y}}}{\partial z}\right)\right] \\
&+ \gamma_{\mathcal{FF}}\left[\frac{1}{c}E_{\mathrm{x}}\left(\mathbf{B}\cdot\frac{\partial \mathbf{B}}{\partial t}\right) + B_{\mathrm{y}}\left(\mathbf{B}\cdot\frac{\partial \mathbf{B}}{\partial z} - \mathbf{E}\cdot\frac{\partial \mathbf{E}}{\partial z}\right) - B_{\mathrm{z}}\left(\mathbf{B}\cdot\frac{\partial \mathbf{B}}{\partial y} - \mathbf{E}\cdot\frac{\partial \mathbf{E}}{\partial y}\right)\right] \\
&+ \gamma_{\mathcal{GG}}\left[-\frac{1}{c}B_{\mathrm{x}}\left(\mathbf{E}\cdot\frac{\partial \mathbf{B}}{\partial t}\right) + E_{\mathrm{y}}\left(\mathbf{E}\cdot\frac{\partial \mathbf{B}}{\partial z} + \mathbf{B}\cdot\frac{\partial \mathbf{E}}{\partial z}\right) - E_{\mathrm{z}}\left(\mathbf{E}\cdot\frac{\partial \mathbf{B}}{\partial y} - \mathbf{B}\cdot\frac{\partial \mathbf{E}}{\partial y}\right)\right]
\end{aligned}$$

Alawashra, Benáček, Pohl, Medvedev
(arXiv:2503.14387, submitted)

Validation and results

1D3V PIC simulation with super-strong magnetic fields

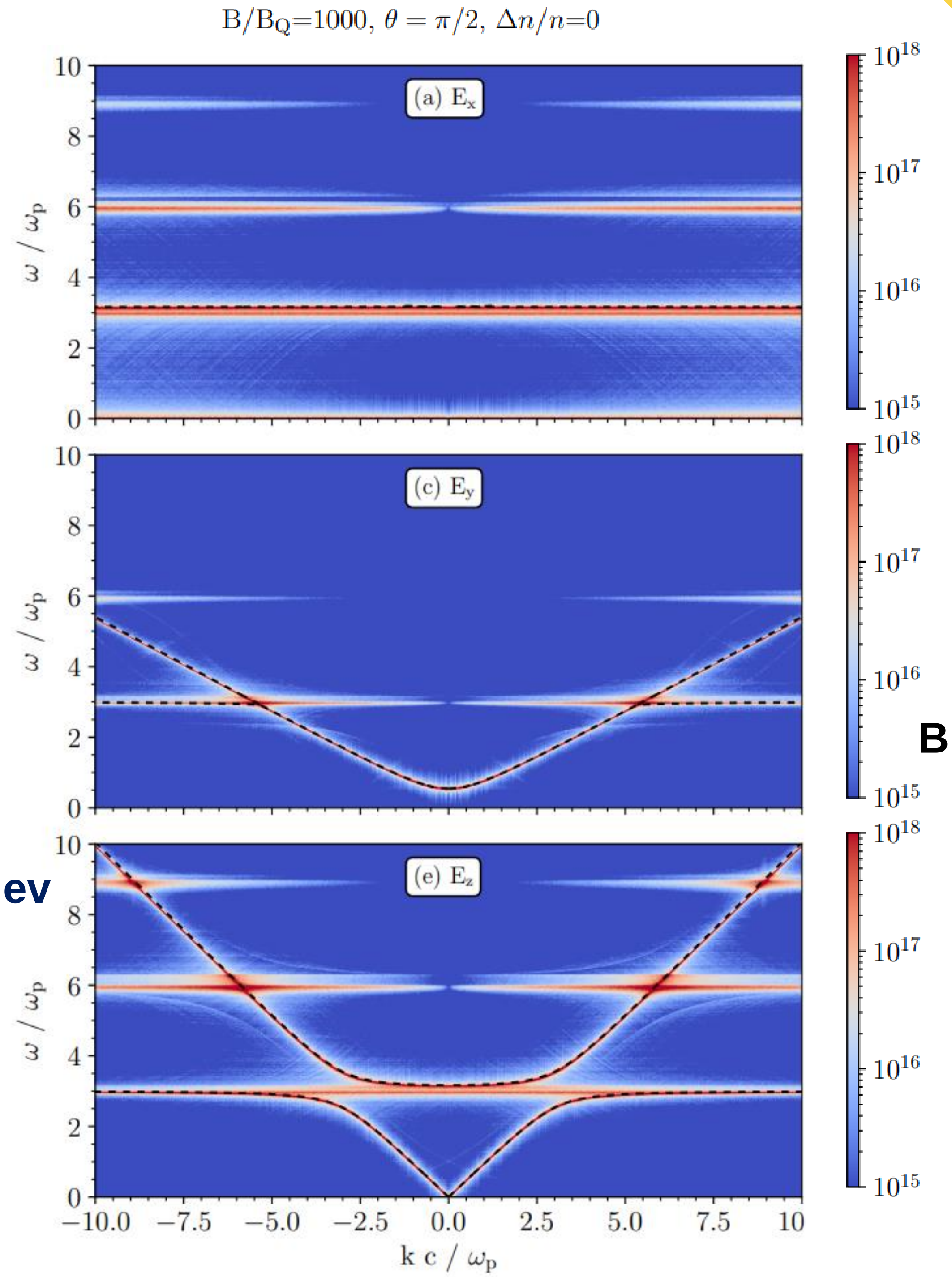
Simulation parameters

Parameter	Value
Magnetic field intensities B/B_{Q}	100, 1000, and 10 000
Magnetic field angles θ	$\pi/30$, $\pi/4$, and $\pi/2$
Frequency ratio $\omega_{\mathrm{c}}/\omega_{\mathrm{p}}$	3
Initial thermal velocity v_{t}/c	0.05
Simulation length L/Δ	20 000
Simulation time $\omega_{\mathrm{p}}t_{\mathrm{end}}$	800
Skin depth resolution Δ/d_{e}	0.05
Time step size $\omega_{\mathrm{p}}\Delta t$	0.02

Our tests show a good agreement with analytical solution

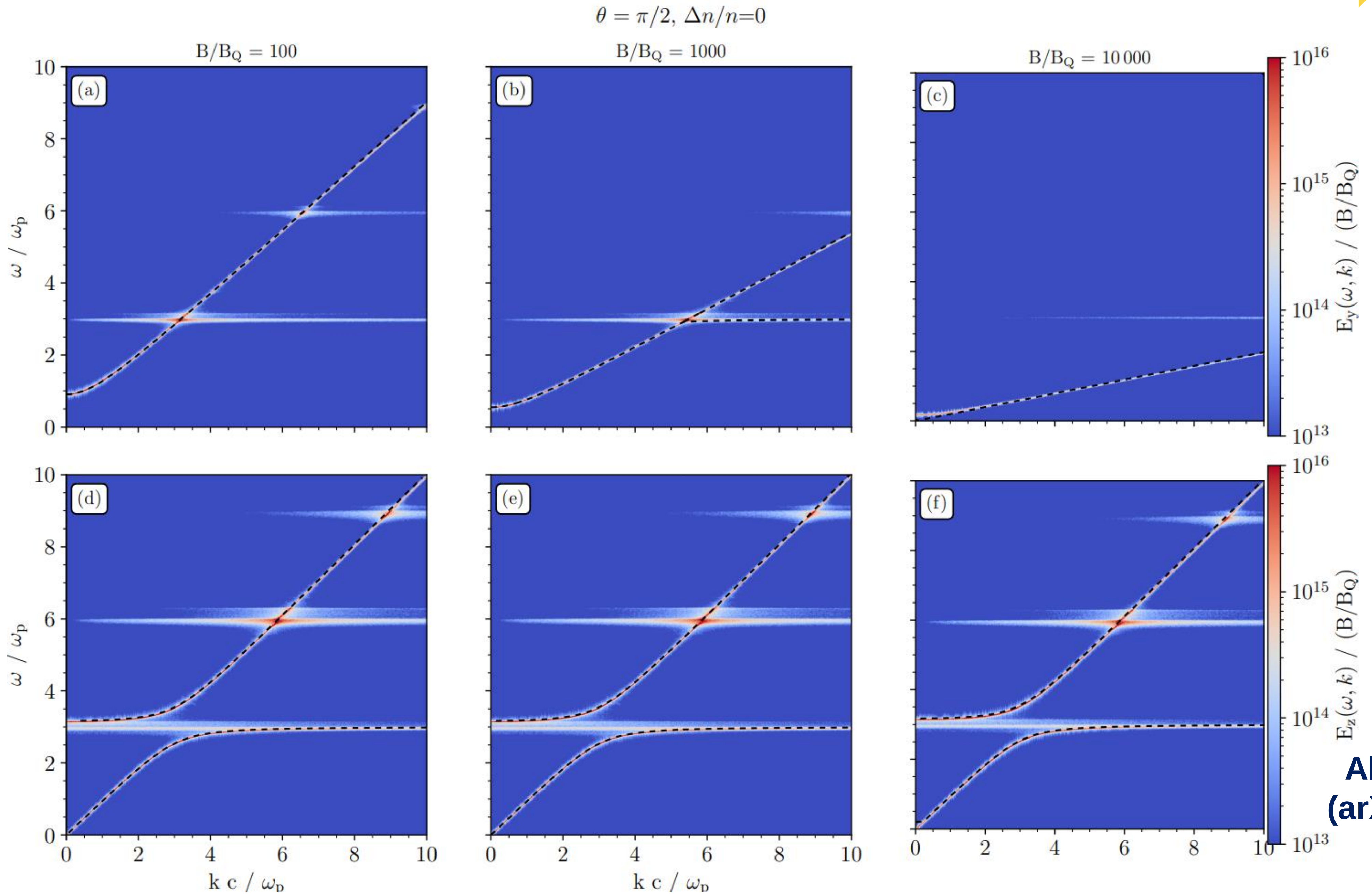
→Plasma birefringence

Alawashra, Benáček, Pohl, Medvedev (arXiv:2503.14387, submitted)



Black line: analytical solution
M. V. Medvedev (2023)

Magnetic field strength dependence



Alawashra et al. (arXiv:2503.14387, submitted)

SUMMARY

- We presented **the first QED polarization PIC solver**, which allows systematic studies of plasma systems with super-critical magnetic fields.
- QED corrections introduce **birefringence between the O- and X-mode** polarized waves with increasing magnetic field intensity.
- This new numerical tool will be improved and applied to model electromagnetic fields in **magnetar magnetospheres**.

Thank you