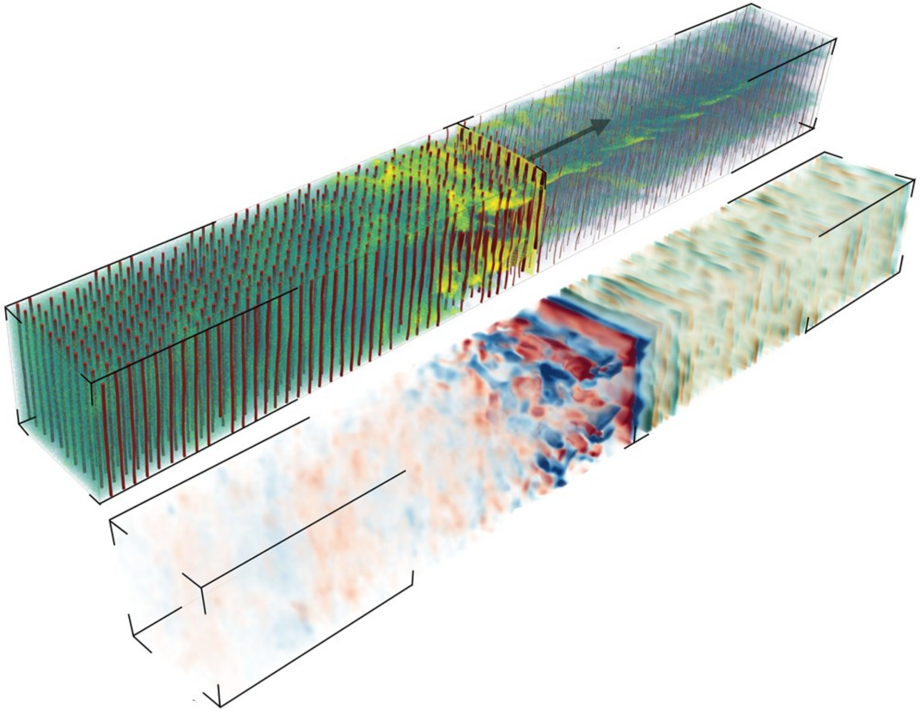
Wave Physics of Fast Radio Bursts



Feeling the pull and the pulse of relativistic magnetospheres,
Les Houches, 05.04.2025.

Ethan van Woerkom
The University of Helsinki

Image:
J. Nättilä

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Magnetar Star Quake Hypothesis

1. Magnetar starquake launches ~kHz frequency Alfvén/FMS Wave

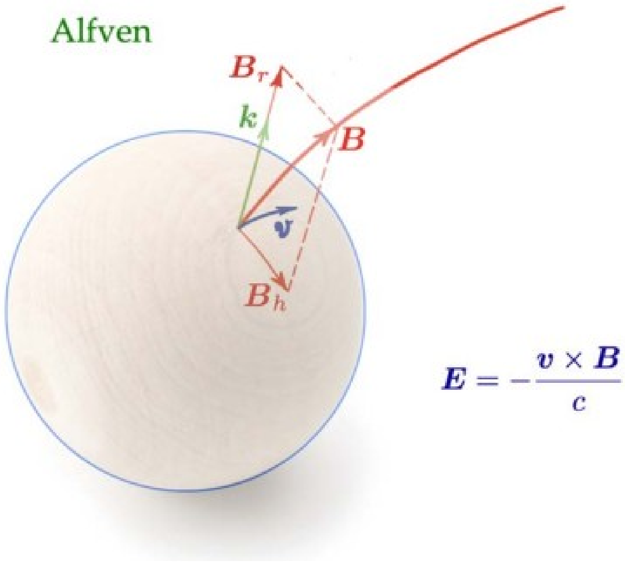
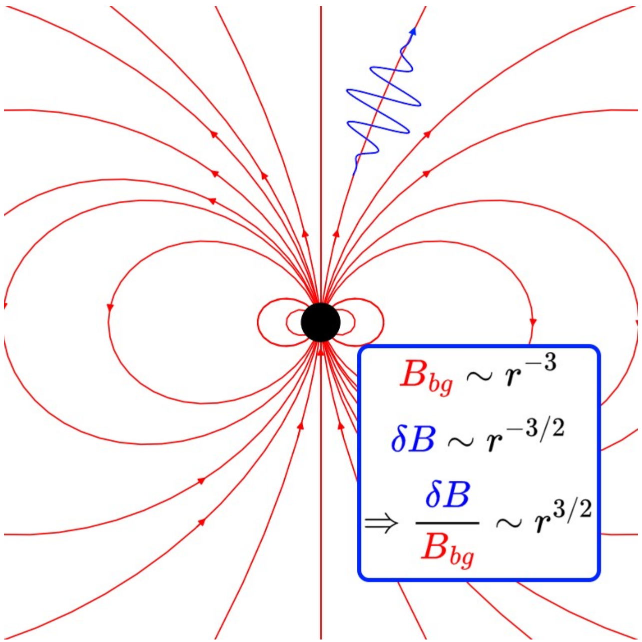


Image: Beloborodov (2023)

2. Alfvén/FMS Wave travels out and becomes nonlinear



Magnetar Star Quake Hypothesis

3. The wave steepens into a shock and triggers coherent emission of higher frequency ~GHz radio waves, e.g. through the synchrotron maser effect

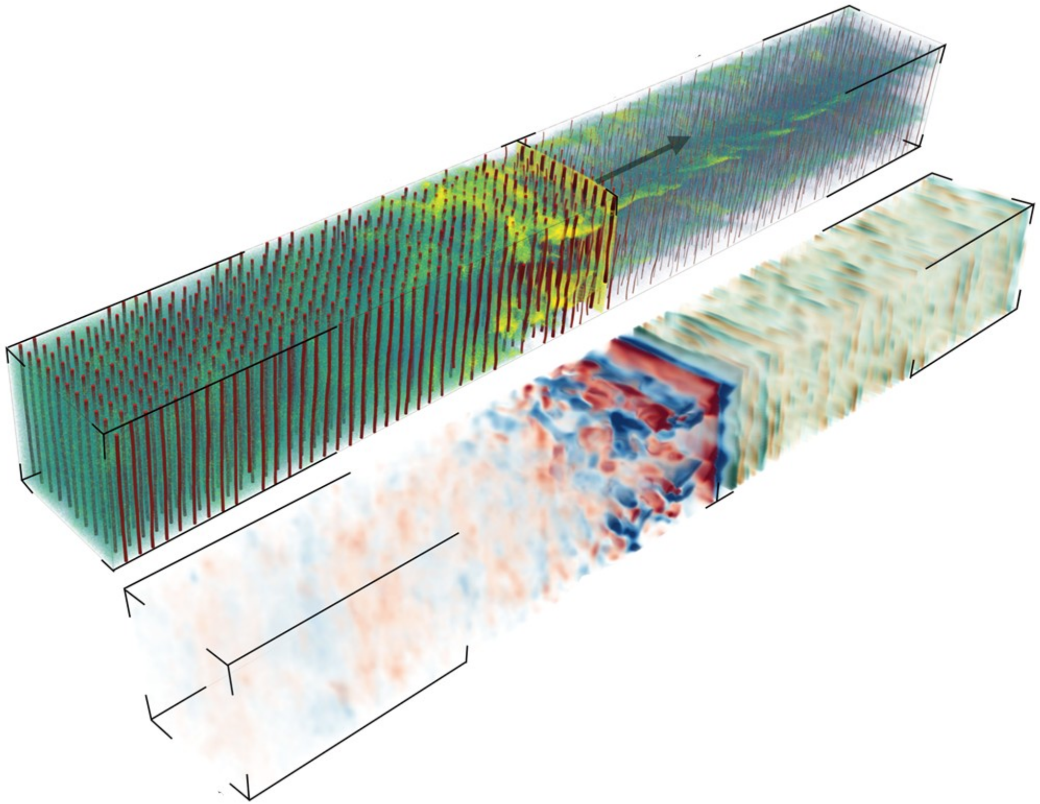


Image:
J. Nättilä

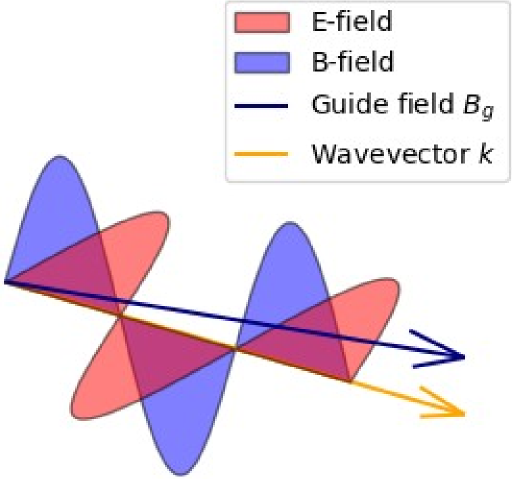
Waves in Ultra-Magnetised Plasmas

Fast Magnetosonic Wave (FMS)

$$\boldsymbol{B}_g \cdot \boldsymbol{B}_w \neq 0$$

$$\boldsymbol{k} \cdot \boldsymbol{E}_w = 0$$

$$v_w = c$$

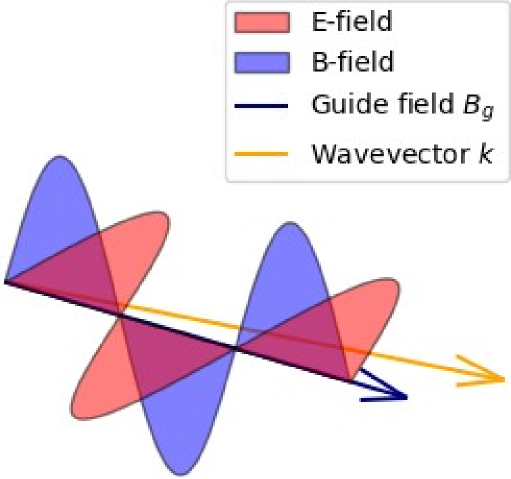


Alfvén Wave

$$\boldsymbol{B}_g \cdot \boldsymbol{B}_w = 0$$

$$\boldsymbol{k} \cdot \boldsymbol{E}_w \neq 0$$

$$v_w = c\cos\theta$$



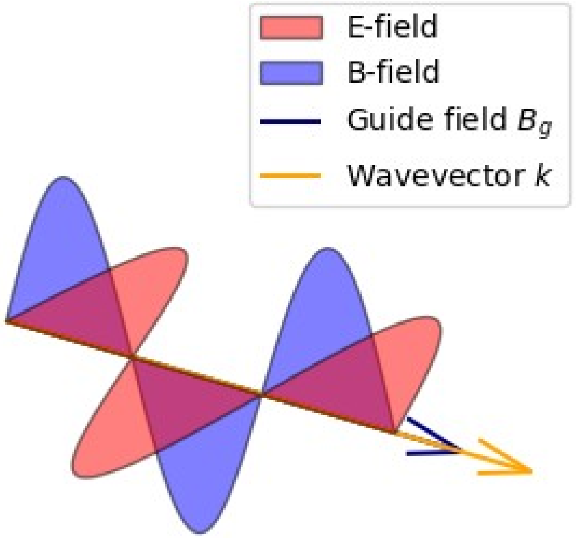
- **Both FMS and Alfvén waves** can be excited in magnetar starquakes. See e.g. Yuan et al. (2020).

Waves in Ultra-Magnetised Plasmas

Parallel Alfvén Wave

$$\boldsymbol{k} \parallel \boldsymbol{B}_g,$$

$$\boldsymbol{B}_g \perp \boldsymbol{B}_w \perp \boldsymbol{E}_w.$$

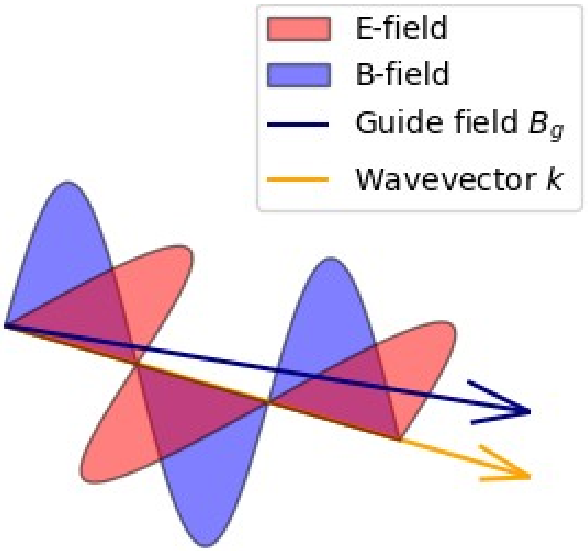


- FMS and Alfvén modes **become identical** if wave travels along guide field.
- Looks like an EM wave super-imposed on a field line.

Waves in Ultra-Magnetised Plasmas

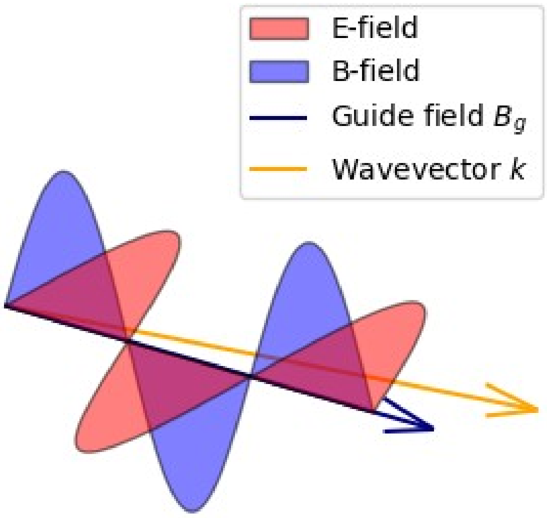
Fast Magnetosonic Wave (FMS)

- Can rapidly induce "monster shocks" once $|\boldsymbol{B}_w| > |\boldsymbol{B}_g|$ occurs



Alfvén Wave

- Cannot induce $|\boldsymbol{B}| > |\boldsymbol{E}|$ even if $|\boldsymbol{B}| \gg |\boldsymbol{E}|$



See Beloborodov (2023), Vanthieghem et al. (2025)

Waves in Ultra-Magnetised Plasmas

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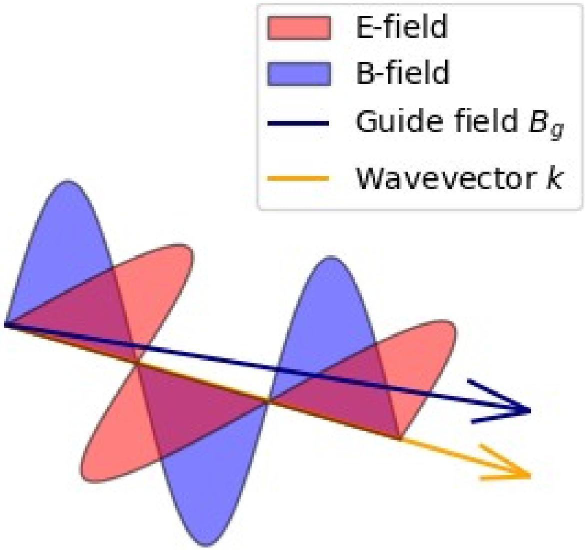

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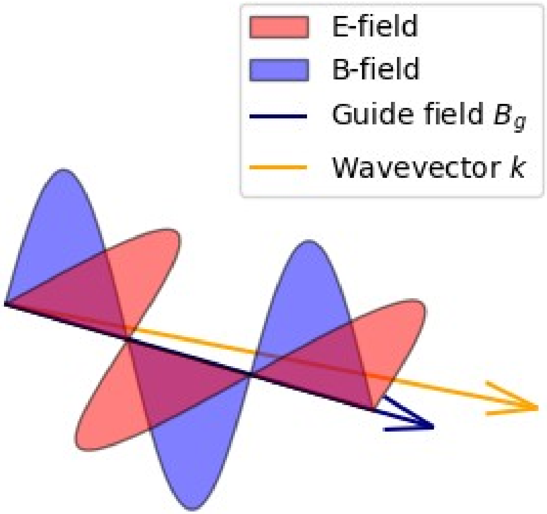
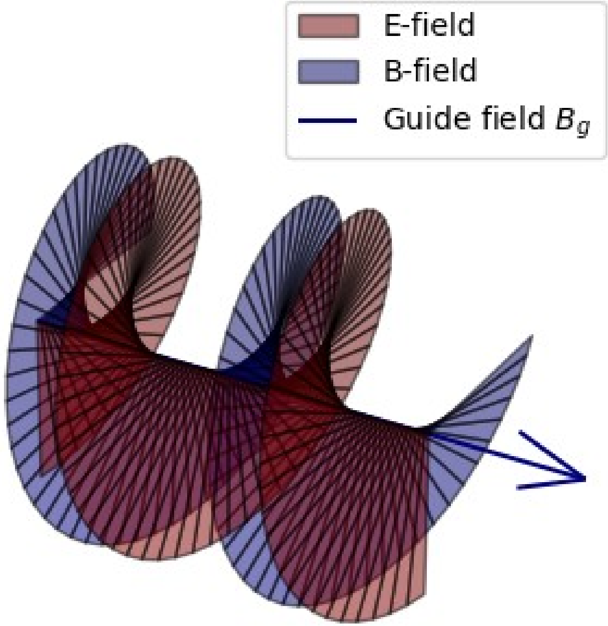


See Beloborodov (2023), Vanthieghem et al. (2025)

Can Alfvén Waves still cause shocks?

Circularly Polarised Alfvén Wave

- Cannot steepen due to symmetry



See Mallet et al. (2023)

Can Alfvén Waves still cause shocks?

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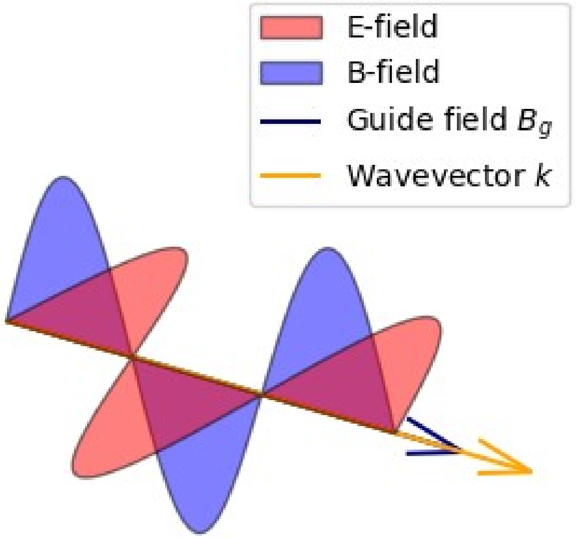


How to tackle this problem properly?

- **Low-frequency waves:**
  - Can be studied within the framework of *Relativistic Magnetohydrodynamics (RMHD)*
- **Ultra-Magnetised plasma:**
  - Solutions will be similar to as in Force Free Electrodynamics (FFED) (Uchida 1997, Kommissarov 2002).
- **Particle-in-cell Simulations**
  - ***Only necessary to study the local shock structure, not necessary for studying shock formation as it is a global process.***

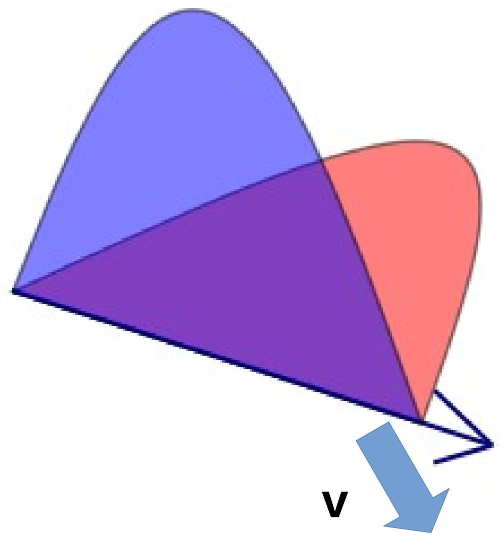
Alfvén Wave Steepening: *Gedanken Experiment*

- Suppose pure E cross B motion of particles through an EM wave super-imposed on magnetic field line.
- We pass particles one-by-one through this wave

$$v = c\frac{\boldsymbol{E}\times\boldsymbol{B}}{|\boldsymbol{B}|^2} = \frac{c}{1+f^2}\begin{pmatrix} f^2 \\ 0 \\ -f \end{pmatrix},$$

$$\gamma = \sqrt{1+f^2},$$

$$f = |\boldsymbol{B}_w| \;/\; |\boldsymbol{B}_g|.$$



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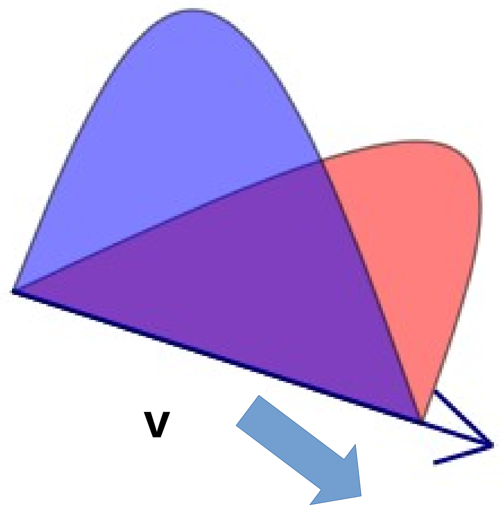

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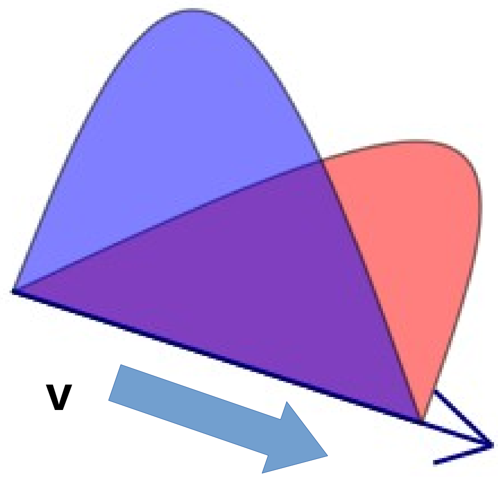

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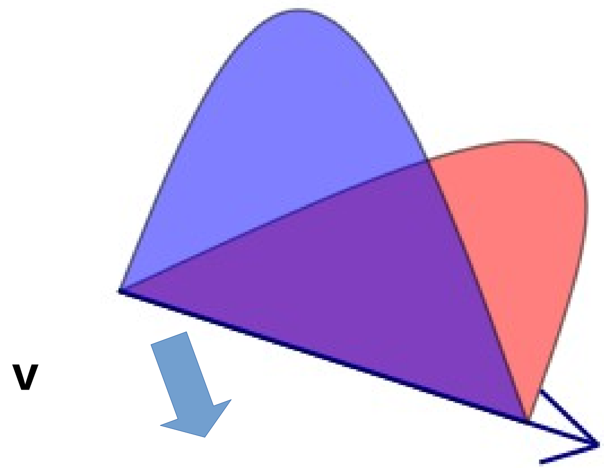
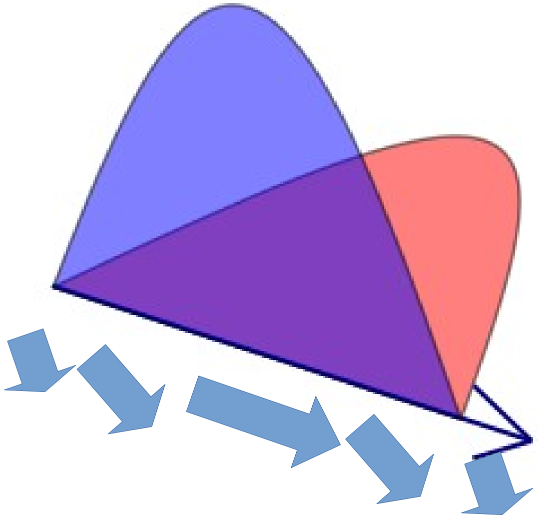
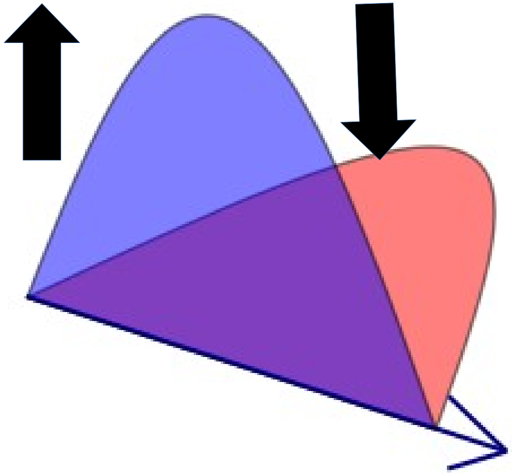


Alfvén Wave Steepening: *Gedanken Experiment*

- In this way, electromagnetic energy is taken from front of wave and deposited in the back.
- Wave amplitude will decrease at front, decrease at back
- Wave “falls over” and steepens into a shock

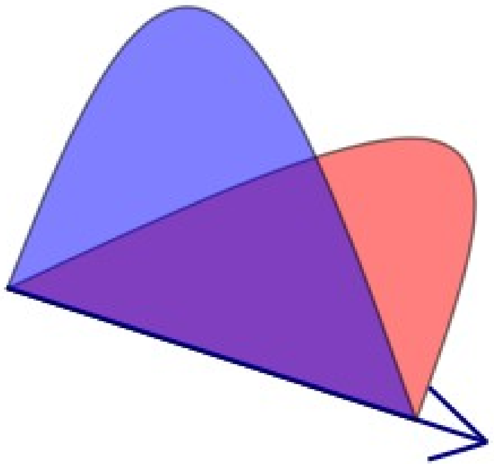




Wave Steepening Distance: Upper Bound

- For peak to catch-up with trough, wave must travel:

$$L = \frac{\text{Wave Energy}}{\text{Energy lost per meter travelled}} = \frac{f^2 \sigma_g \rho_0 c^2 W}{\rho_0 c^2 \left(\sqrt{1+f^2} - 1\right)} \approx \sigma_g f W$$



One Flaw In This Analysis

- The E cross B motion is not pure: we neglected motion parallel to the magnetic field.

$$\frac{Dp_{\|}}{Dt} = \frac{D}{Dt}\left(\boldsymbol{p}_{\text{fluid}} \cdot \hat{B}\right)$$
$$= (\nabla \cdot v)p_{\|} - \left(\hat{B} \cdot \nabla\right)p_{\|} + \hat{c} \cdot \boldsymbol{p}_{\perp}.$$

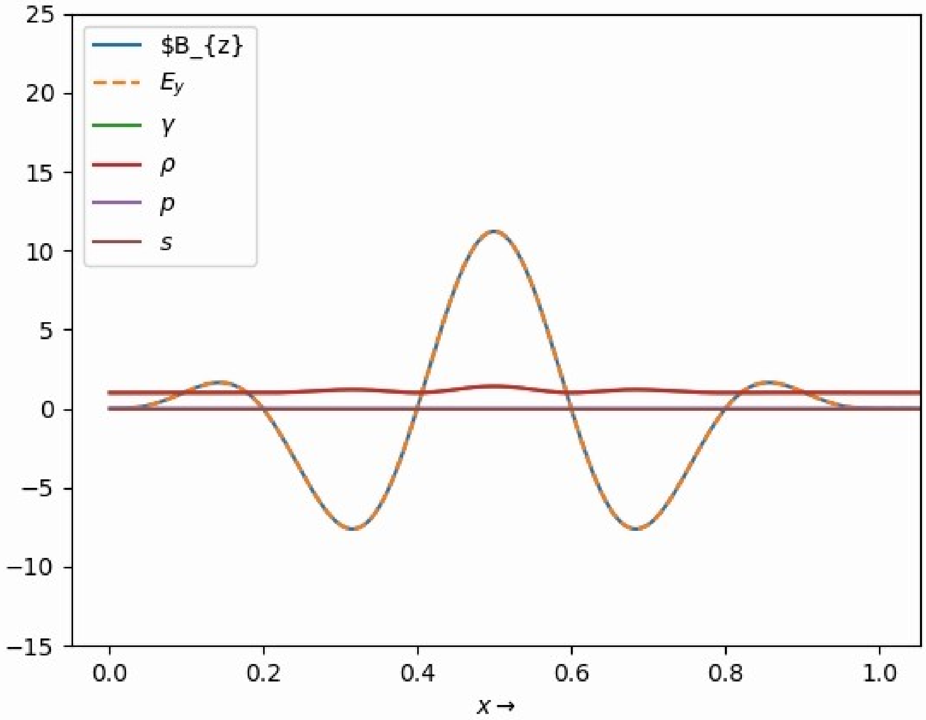
- Fully capturing this effect requires a complete analysis of RMHD equations, more on this later.

RMHD Simulation with AthenaK

- Used AthenaK RMHD code (Stone et al. 2024)
- Set up a 1D problem with nonlinear parallel Alfvén wavepacket.
- Pressure: 1% of rest mass density, EOS: $\gamma = 4/3$.
- “Race track” of 200 wavepackets long.
- 131072 cells, 6 AMR Levels

RMHD Simulation (1)

Mildly Nonlinear: σ = 10, f = $|B_w|/|B_g|$ = 1.



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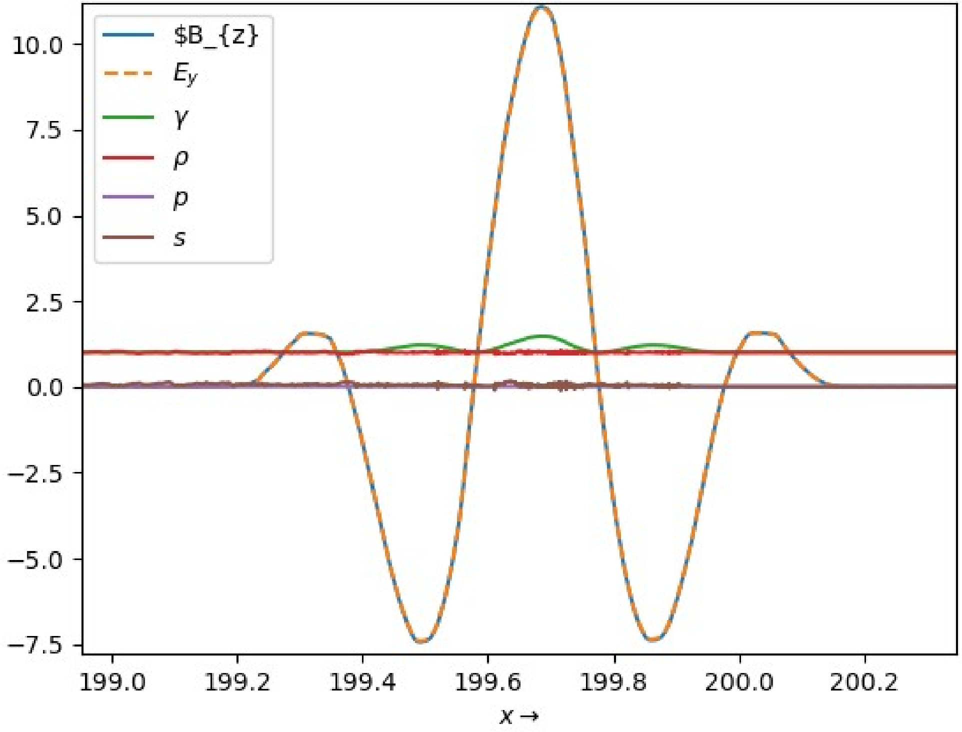
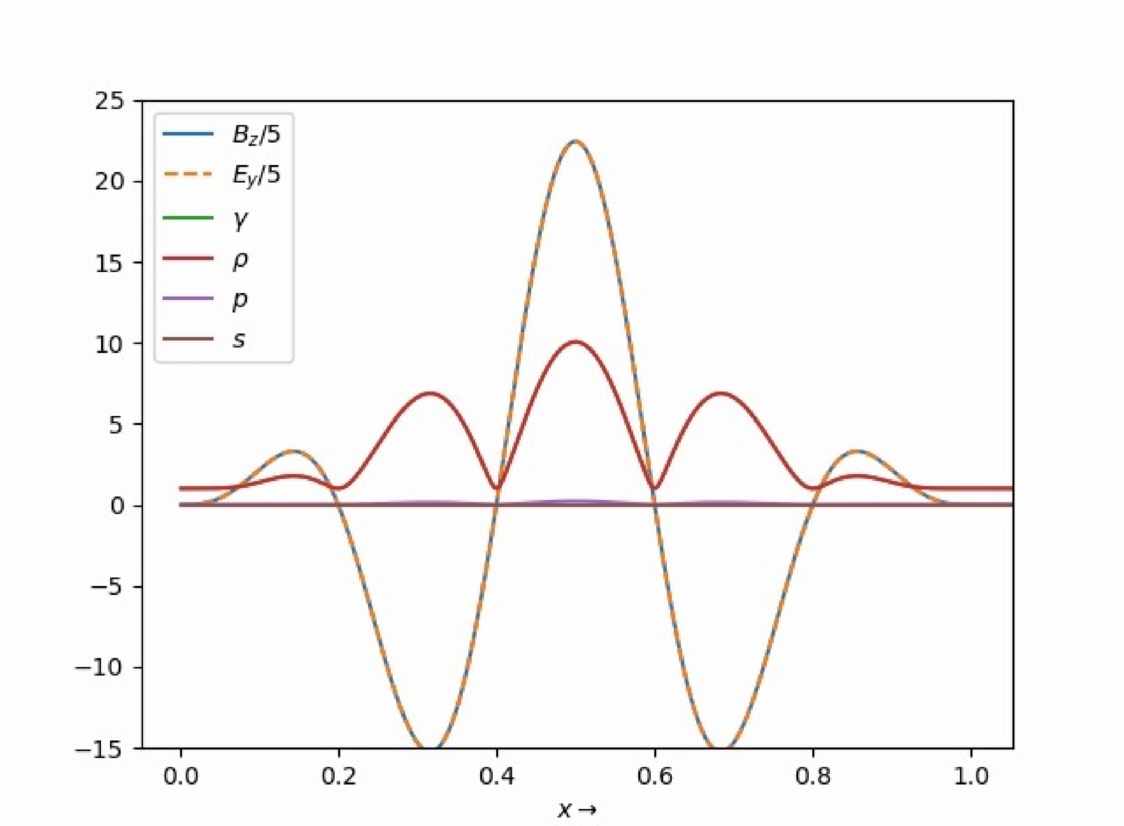


RMHD Simulation (2)

Strongly Nonlinear: σ = 10, f = $|B_w|/|B_g|$ = 10.



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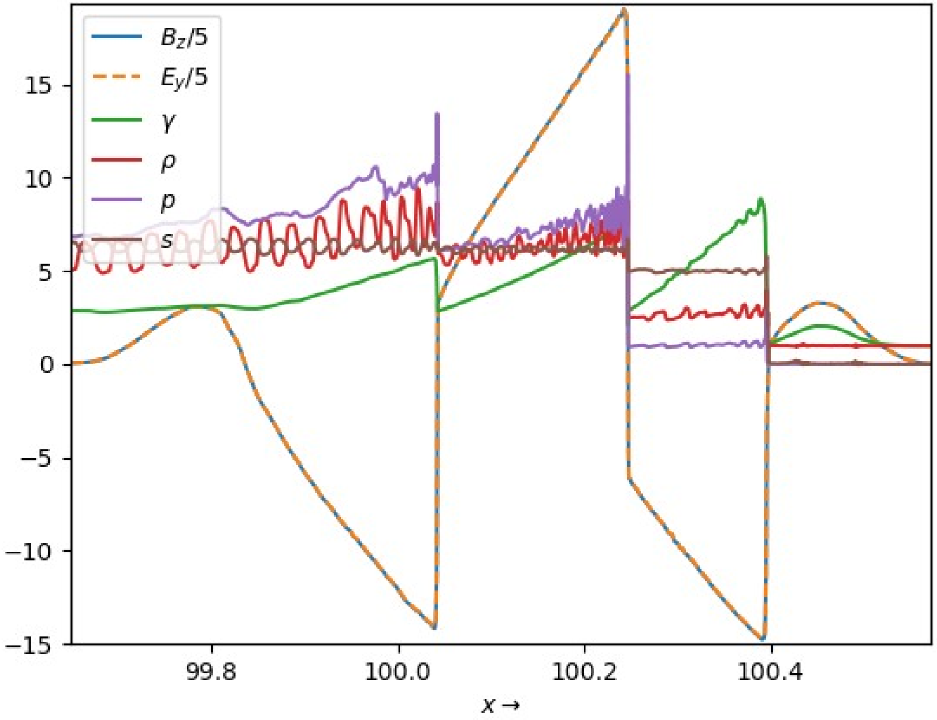
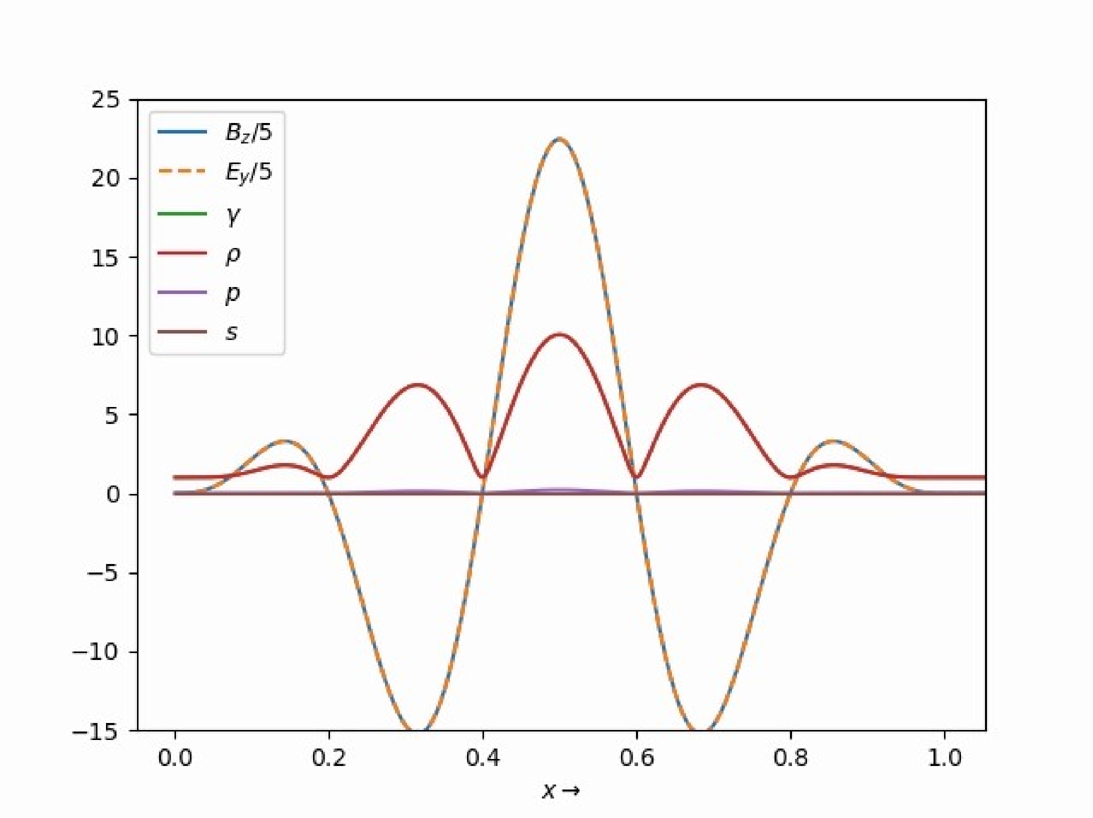


RMHD Simulation (3)

More Magnetised: $\sigma = 20$, $f = |B_w|/|B_g| = 10$.



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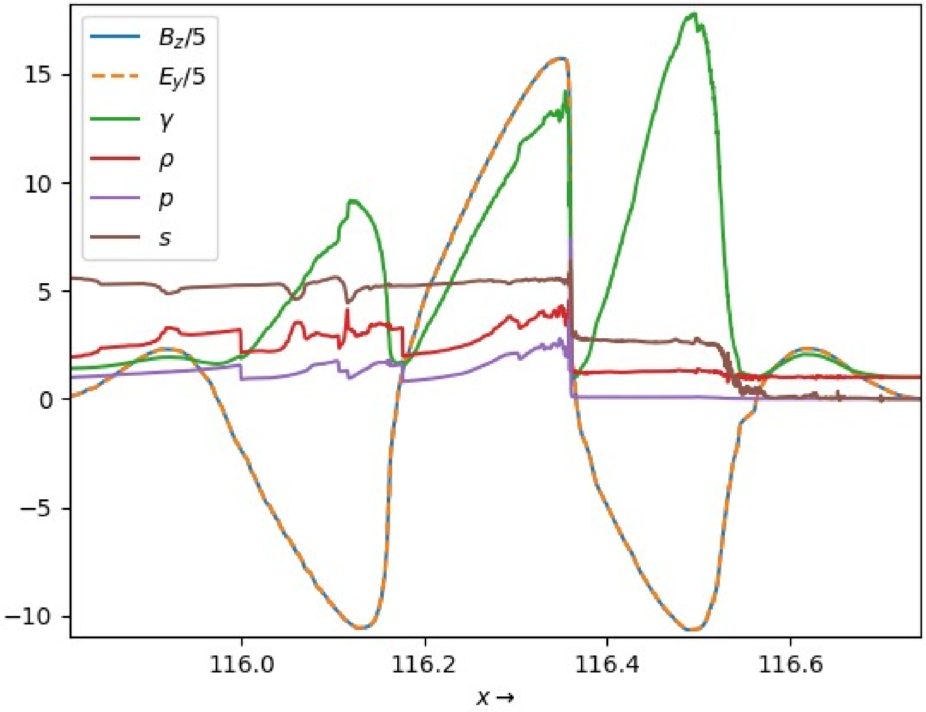

Parallel Alfvén waves *can steepen into shocks*

- We have developed an analytical treatment describing this process in the limit **$\sigma >> 1$: Woerkom & Nättilä (in prep.)**
- This treatment can be extended to oblique Alfvén waves and fast magnetosonic waves.

  Question:

  The shock discontinuity will be of type "**Alfvén Shock"**, cf. Lichnerowicz (1967).

  Why is the literature essentially silent on Alfvén shocks?